Taylor Rules with Headline Inflation: A Bad Idea!*

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June 13, 2007

Abstract

Should a central bank accommodate energy price shocks? Should the central bank use core inflation or headline inflation with the volatile energy component in its Taylor rule? In order to answer these questions we build a dynamic stochastic general equilibrium (DSGE) model with energy use, durable goods and nominal rigidities to study the effects of an energy price shock and its impact on the macroeconomy when the central bank follows a Taylor rule. We then study how the economy performs under alternative parameterizations of the rule with different weights on headline and core inflation after an increase in the energy price. Our simulation results indicate that a central bank using core inflation in its Taylor rule does better than one using headline inflation because the output drop is less severe. In general, we show that the lower the weight on energy price inflation in the Taylor rule, the lower the impact is of an energy price increase on GDP and inflation.

JEL codes: E32, Q43

Keywords: energy prices, business cycles, durable goods, nominal rigidities, monetary transmission mechanism

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*We thank seminar participants at the Federal Reserve Bank of Atlanta for their helpful comments. All errors are our responsibility. The views expressed here are not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System.

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1 Introduction

Should a central bank accommodate energy price shocks? Should the central bank use core inflation or headline inflation with its volatile energy component if it follows a Taylor rule? In order to answer these questions we study a dynamic stochastic general equilibrium (DSGE) model with energy price shocks in the presence of money, nominal rigidities and durable goods investment. Additionally, this model has energy use at both the firm and the household level as in Dhawan and Jeske (2006). We introduce a generalized Taylor rule that explicitly distinguishes between core and energy price inflation. We use this model to study the effects of an energy price shock on the economy in the presence of alternative parameterizations of the monetary policy rule.

We find that the central bank cannot completely shield the economy from an energy price spike. However, a central bank using core inflation explicitly in its Taylor rule does better than the one using headline inflation because the output drop is less severe. In general, we show that the lower the weight on the energy price inflation in the Taylor rule, the lower the impact is of energy price shocks on GDP and inflation. This result appears contrary to conventional wisdom in monetary economics, whereby a policy that accommodates oil price shocks is actually counter-productive, as in Leduc and Sill (2004) who state that "[e]asy inflation policies are seen to to amplify the impacts of oil price shocks on output and inflation"(p. 806). However, our result is not contradicting the previous research. In fact, we replicate the results in Leduc and Sill (2004) that with low levels of nominal rigidities, a policy rule with more weight on the output gap or a lower weight on the core inflation exacerbates the output drop and the inflation spike following an energy price hike.[1] We simply show that the central bank can "accommodate" inflation as long as that refers to energy price inflation only.

Our results also vindicate the work of Bernanke, Gertler and Watson (1997) who claim that the Federal Reserve should have been less aggressive in responding to energy price hikes in the 1970s, which would have stabilized inflation without jeopardizing economic growth. Their paper was criticized for its reduced-form vector auto-regressions (VARs) methodology, that is subject to the Lucas Critique because any change in the monetary policy rule would trigger a change in the underlying parameters of that VAR model.[2] Our model, however, shows that even within a rational expectations framework the central bank can indeed accommodate an inflationary shock as long as the accommodation comes in the form of low or even negative weights on the energy price inflation while staying vigilant on core inflation.

[1] However, we do show that with more wage and price rigidity we can reverse that result. Specifically, the output drop under easy monetary policy (lower weight on inflation or higher weight on the output gap) can cushion the output drop.

[2] Zha (1999) and Hamilton and Herrera (2004) also criticize their VARs for misspecification.

Our results are similar in spirit to the work of Carlstrom and Fuerst (2006a) who show that temporarily accommodating an energy price hike softens the output drop. This policy is not counter-productive the way Leduc and Sill (2004) showed, because the the central bank leaves the coefficients in the Taylor rule unchanged but temporarily deviates from the rule. The difference between our work and Carlstrom and Fuerst is that we incorporate the accommodation *explicitly* through an additional term for energy price inflation in the Taylor rule.[3]

Following Leduc and Sill (2004), the nominal rigidities on the consumer side are modeled in the form of a cash in advance constraint with adjustment costs for changing nominal wages. On the producer side, firms in the intermediate goods sector face adjustment costs for their nominal price. Also, firms have to borrow funds to finance their payroll. A major difference from Leduc and Sill's specification is that we introduce durable goods investment and energy use at the household level as in Dhawan and Jeske (2006). Another feature of our model is that we introduce energy use directly in the production function where capital and energy are complements, as in Kim and Loungani (1992) and Carlstrom and Fuerst (2006a), unlike Leduc and Sill (2004) who tie energy use to capacity utilization.

As in Dhawan and Jeske (2006), the drop in GDP after an energy price hike is smaller in an economy with durable goods than in one without them. This is due to a portfolio rebalancing effect whereby the representative consumer lowers durables investment more than fixed investment, which cushions the drop in output. We can see a flavor of this rebalancing action in Figure 1, which plots empirical impulse response functions (IRFs) after a one standard deviation shock to the energy price.[4] Durable goods investment drops immediately after the energy price shock while fixed investment actually increases slightly for a while and then drops and by a lower magnitude than durable goods investment.[5] We show that this rebalancing effect plays a major role in our results. Specifically, we show that a Taylor rule with headline inflation impedes this rebalancing and thus causes larger output drops. In contrast, if the central bank puts a low (potentially negative) weight on energy inflation it enhances the rebalancing and thus cushions the drop in output.

The paper proceeds as follows. Section 2 outlines the model, section 3 goes through the calibration and estimation used to parameterize our model, section 4 shows the numerical results, both in the baseline economy as well as under different policy rules and section 5 concludes.

[3]This idea of a generalized Taylor rule is similar to the work of Bernanke and Gertler (1999, 2001) who add an extra term for the asset price change to the Taylor rule to study the role of monetary policy in an economy with asset price shocks.

[4]We use quarterly data from 1970:1 to 2006:4 from the BEA. Core inflation refers to changes in the deflator of personal consumption expenditures (PCE) outside of energy. The energy price refers to the price of PCE energy relative to non-energy PCE goods and services.

[5]This delayed drop in fixed investment apparent in the empirical IRFs is also captured in the model of Dhawan and Jeske (2006).

2 Model

2.1 Households

There is a measure one of households indexed by $i \in [0,1]$. Households have preferences over consumption and hours given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_{i,t}^A, H_{i,t}\right) \tag{1}$$

where $C_{i,t}^A$ is a consumption aggregator and $H_{i,t}$ are hours worked. C^A consists of three goods; nondurables and services excluding energy (N), the flow of services from the stock of durables goods (D) and energy use (E_h). We write the period t utility function as following:

$$u\left(C_{i,t}^A, H_{i,t}\right) = \varphi \log C_{i,t}^A + (1-\varphi)\log(1-H_{i,t}) \tag{2}$$

where $\varphi \in (0,1)$ and H denotes hours worked. This log-utility specification is the same as in Kim and Loungani (1992).

As in Dhawan and Jeske (2006) we choose the following functional form for the aggregator function to combine these three types of consumption into C^A:

$$C_{i,t}^A = N_{i,t}^{1-\gamma}\left(\eta_h D_{i,t-1}^{\nu_h} + (1-\eta_h) E_{h,i,t}^{\nu_h}\right)^{\frac{\gamma}{\nu_h}} \tag{3}$$

Notice the timing of the durables stock. We index each variable by the time period its level was set. The stock of durables evolves according to:

$$D_{i,t} = (1-\delta_d) D_{i,t-1} + I_{d,i,t} \tag{4}$$

where $I_{d,i,t}$ denotes durables investment. Households face an adjustment cost when changing durable goods investment:

$$AC_{j,t}^d = \frac{\phi_d}{2}\left(\frac{I_{d,i,t}}{D_{i,t-1}} - \delta_d\right)^2 I_{d,i,t} \tag{5}$$

In the labor market households are monopolistically competitive. Total labor services H_t available to the production sector are aggregated across households via:

$$H_t = \left(\int_0^1 H_{i,t}^{(\theta_w-1)/\theta_w} di\right)^{\theta_w/(\theta_w-1)} \tag{6}$$

As usual this generates a downward sloping demand curve for household i labor services:

$$H_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} H_t \tag{7}$$

where $W_{i,t}$ is the household specific wage and W_t aggregates the individual wages via:

$$W_t = \left(\int_0^1 W_{i,t}^{1-\theta_w} di\right)^{1/(1-\theta_w)} \tag{8}$$

Alternatively we can write equation (7) as:

$$W_{i,t} = \left(\frac{H_{i,t}}{H_t}\right)^{-1/\theta_w} W_t \tag{9}$$

Thus real labor income is:

$$\begin{aligned} H_{i,t}W_{i,t} &= H_{i,t}\left(\frac{H_{i,t}}{H_t}\right)^{-1/\theta_w} W_t \\ &= W_t H_t^{1/\theta_w} H_{i,t}^{1-\theta_w} \end{aligned} \tag{10}$$

Let P_t be the core price level in period t and $\pi_t = P_t/P_{t-1}$ be the gross core inflation rate. Households face an adjustment cost $AC^w$for changing the nominal wage:

$$AC_{i,t}^w = \frac{\phi_w}{2}\left(\pi_t \frac{W_{i,t}}{W_{i,t-1}} - \bar{\pi}\right)^2 W_{i,t} \tag{11}$$

where $\bar{\pi}$ is steady state inflation. Households begin every period with $M_{i,t-1}$ dollars carried over from last period. They make a deposit of $DP_{i,t}$ at the intermediary and use the remaining money balance to finance all consumption expenditures. This induces the cash in advance constraint:

$$N_{i,t} + I_{d,i,t} + P_t^e E_{h,i,t} \leq \frac{M_{i,t-1}}{P_t} - \frac{DP_{i,t}}{P_t} \tag{12}$$

where P_t^e is the relative price of energy. Notice that our definition of core price is slightly different from what is normally used in that the food component included in N is part of our core price. Food prices, of course, are excluded in the both the core PCE deflator and the core CPI index. Since most of the variance in headline inflation is due to energy rather than food price fluctuations, however, we argue that our core index is a good enough approximation for the real world core price index.[6]

[6]For example, the correlation between PCE ex food and energy and PCE ex energy was 0.9637 between 1970Q1 and 2006Q4. What's more, food prices have become less volatile over time: between the years 2000 and 2006, food price fluctuations account for only about 1.4 percent of the variation in the difference between core and headline inflation.

Real money holdings evolve according to:

$$\begin{aligned}\frac{M_{i,t}}{P_t} &= \frac{M_{i,t-1}}{P_t} - \frac{DP_{i,t}}{P_t} - \left(N_{i,t} + I_{d,i,t} + P_t^e E_{h,i,t}\right) - \left(AC_{i,t}^w + AC_{i,t}^d\right) \\ &\quad + R_t \frac{DP_{i,t}}{P_t} + W_{i,t} H_{i,t} + \Pi_{i,t}^f + \frac{\Pi_{i,t}^b}{P_t} \qquad (13)\end{aligned}$$

where $\Pi_{i,t}^f$ is the real dividend from the firm and $\Pi_{i,t}^b$ is the nominal dividend from the financial intermediary. Also notice that the deposits that the household made at the beginning of the period pay off at the end of the period, including interest. They add to the end-of period money stock.

We can now write the consumer problem as:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_{i,t}^A, H_{i,t}\right)$$

subject to:

$$\begin{aligned}\frac{M_{i,t}}{P_t} &= \frac{M_{i,t-1}}{P_t} - \frac{DP_{i,t}}{P_t} - \left(N_{i,t} + I_{d,i,t} + P_t^e E_{h,i,t}\right) - AC_{i,t}^w - AC_{i,t}^d \\ &\quad + R_t \frac{DP_{i,t}}{P_t} + W_t H_t^{1/\theta_w} H_{i,t}^{1-\theta_w} + \Pi_{i,t}^f + \frac{\Pi_{i,t}^b}{P_t} \qquad (14) \\ N_t + I_{d,i,t} + P_t^e E_{h,i,t} &\leq \frac{M_{i,t-1}}{P_t} - \frac{DP_{i,t}}{P_t} \qquad (15) \\ C_{i,t}^A &= N_{i,t}^{1-\gamma}\left(\eta_h D_{i,t-1}^{\nu_h} + \left(1-\eta_h\right) E_{h,i,t}^{\nu_h}\right)^{\frac{\gamma}{\nu_h}} \qquad (16) \\ D_{i,t} &= \left(1-\delta_d\right) D_{i,t-1} + I_{d,i,t} \qquad (17)\end{aligned}$$

2.2 Financial Intermediary

As in Leduc and Sill (2004), the financial intermediary takes deposits from households, receives money injections X_t from the central bank and loans funds to the firm. That way the money injection including interest is passed on to households as the dividend:

$$R_t X_t = \int_0^1 \Pi_{i,t}^b di$$

2.3 Firms

We assume that all goods are produced in one single sector.[7] The final good Y_t is produced in a competitive market by aggregating intermediate goods $Y_{j,t}$ of a measure one of monopolistically competitive firms indexed by $j \in [0,1]$. The aggregator over intermediate goods is:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\left(\theta_f - 1\right)/\theta_f} dj \right)^{\theta_f/\left(\theta_f - 1\right)} \tag{18}$$

The aggregate price level is:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta_f} dj \right)^{1/\left(1-\theta_f\right)} \tag{19}$$

and the demand for firm j output is:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_f} Y_t \tag{20}$$

where $P_{i,t}$ is the intermediate goods price and P_t aggregates the individual prices via:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta_f} dj \right)^{1/\left(1-\theta_f\right)} \tag{21}$$

The price as a function of firm j supply is:

$$P_{j,t} = \left(\frac{Y_{j,t}}{Y_t} \right)^{-1/\theta_f} P_t \tag{22}$$

In what follows, when we talk about firms we mean the intermediate goods producers, not the final output producer. Firms can adjust the price of their output but face an adjustment cost:

$$AC_{j,t}^{p} = \frac{\phi_p}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - \bar{\pi} \right)^2 Y_{j,t} \tag{23}$$

Each firm j produces its output $Y_{j,t}$ with the following production function as in Dhawan and Jeske (2006):

$$Y_{j,t} = Z_t H_{j,t}^{1-\alpha} \left(\eta_f K_{j,t-1}^{\nu_f} + \left(1 - \eta_f\right) E_{f,j,t}^{\nu_f} \right)^{\alpha/\nu_f} \tag{24}$$

where Z is an aggregate shock to productivity, H_j are hours employed in production, $K_{j,t-1}$ is capital and $E_{f,j,t}$ is firm energy use.[8]

[7] Barsky, House and Kimball (2005), Erceg and Levin (2006) and Carlstrom and Fuerst (2006b) use a sticky price model where durable goods are produced in a separate sector. In a two sector model we would be able to study the relative price movement of durables versus non-durable goods after an energy price shock. We refer this to future research.

[8] This setup is different from the model in Aoki (2000) who studies how the central bank should respond to relative price changes. In Aoki's model the good with the flexible price, which he interprets as energy does not

Capital evolves according to:

$$K_{j,t} = (1 - \delta_k) K_{j,t-1} + I_{k,j,t} \tag{25}$$

where $I_{k,j,t}$ stands for firm j's fixed capital investment. Just like households, firms have to pay an adjustment cost to change their level of investment:

$$AC_{j,t}^k = \frac{\phi_k}{2} \left(\frac{I_{k,j,t}}{K_{j,t-1}} - \delta_k \right)^2 I_{k,j,t} \tag{26}$$

Firms borrow money from the intermediary to pay for their wage bill. The total dividend in real terms is then:

$$\begin{aligned} \Pi_{j,t}^f &= Y_{j,t} \frac{P_{j,t}}{P_t} - W_t H_{j,t} R_t - I_{k,j,t} - P_t^e E_{f,j,t} - AC_{j,t}^k - AC_{j,t}^p \\ &= Y_{j,t} Y_t^{1/\theta_f} - W_t H_{j,t} R_t - I_{k,j,t} - P_t^e E_{f,j,t} - AC_{j,t}^k - AC_{j,t}^p \end{aligned} \tag{27}$$

The objective function of the firm is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^B \Pi_{j,t}^f$$

subject to:

$$\Pi_{j,t}^f = Y_{j,t} Y_t^{1/\theta_f} - W_t H_{j,t} R_t - I_{k,j,t} - P_t^e E_{f,j,t} - AC_{j,t}^k - AC_{j,t}^p \tag{28}$$

$$K_{j,t} = (1 - \delta_k) K_{j,t-1} + I_{k,j,t} \tag{29}$$

$$Y_{j,t} = Z_t H_{j,t}^{1-\alpha} \left(\eta_f K_{j,t-1}^{\nu_f} + (1 - \eta_f) E_{f,j,t}^{\nu_f} \right)^{\alpha/\nu_f} \tag{30}$$

The pricing kernel $\beta^t \lambda_t^B$ is the Lagrange multiplier on the consumer budget constraint (in real terms). That is, the firm discounts its profits at the marginal utility of dividends on the consumer side.

2.4 Aggregate Resource Constraint

The aggregate resource constraint is:

$$Y_t = N_t + I_{d,t} + P_t^e E_{h,t} + I_{k,t} + P_{f,t}^e E_{f,t} + AC_t^d + AC_t^w + AC_t^d + AC_t^p \tag{31}$$

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enter the production function.

2.5 Monetary Policy

Monetary policy follows a Taylor rule with interest rate persistence. We model the Taylor rule in a sightly more general way as in Clarida, Gali and Gertler (2000), to accommodate more general interest rate rules. Specifically, we allow the central bank to use not just core inflation $\pi_t = \frac{P_t}{P_{t-1}}$ but also headline inflation in its interest rate rule. Notice that we can express headline CPI as:

$$\pi_t^{HL} = \pi_t + \chi^e \pi_t^e \tag{32}$$

where $\pi_t^e = \frac{P_t^e}{P_{t-1}^e}$ is the price change of the relative energy price and $\chi^e = \frac{P^e E_h}{N + I_d + P^e E_h}$ is the steady state share of of energy expenditures for consumers. Also notice that in the steady state, only the nominal price P_t grows at a positive rate, not the relative energy price P^e, therefore $\bar{\pi}^e = 0$. Then we use the generalized Taylor rule with an additional term for energy price inflation:

$$R_t - \bar{R} = \rho_r \left(R_{t-1} - \bar{R}\right) + (1 - \rho_r) \left[\tau_\pi^{core} \left(\pi_t - \bar{\pi}\right) + \tau_\pi^e \pi_t^e + \tau_y \left(Y_t - \bar{Y}\right)\right] \tag{33}$$

This setup is similar in spirit to Bernanke and Gertler (1999, 2001) who add an extra term for the asset price to study the role of monetary policy in response to asset price shocks. Notice that with this equations we can accommodate a wide variety of Taylor rules. The rule used in Leduc and Sill (2004) is a special case if we set $\tau_\pi^e = 0$. Likewise, if we set $\tau_\pi^{core} = \tau_\pi^{HL}$ and $\tau_\pi^e = \chi^e \tau_\pi^{HL}$ the rule is equivalent to one where the central bank puts weight τ_π^{HL} on headline inflation.

3 Calibration and Estimation

A summary of all estimated and calibrated parameters is in Table 1.

Table 1: Model Parameters

Households		Firms		Central Bank		Shocks	
β	0.9900	α	0.3600	ρ_r	0.7900	$z_{y,t} = \rho_z z_{y,t-1} + \varepsilon_{z,t}$	
θ_w	3.0000	θ_f	11.0000	τ_π^{core}	1.8000	ρ_z	0.9500
ν_h	-3.0000	ν_f	-0.7000	τ_π^e	0.0000	σ_z^2	0.0070
ϕ_w	23.0766	ϕ_p	115.3829	τ_y	0.2700		
ϕ_d	11.6305	ϕ_k	144.9298	$\bar{R}$	0.0150	$p_t = \rho_p^1 p_{t-1} + \varepsilon_{p,t} + \rho_p^2 \varepsilon_{p,t-1}$	
η_h	1.5824×10^{-5}	η_f	0.9955			ρ_p^1	0.9753
δ_d	0.0682	δ_k	0.0107			ρ_p^2	0.4217
γ	0.1831					σ_p^2	0.0308
φ	0.4502						

3.1 Preferences and technology

One model period corresponds to one quarter, so we set the discount rate β at 0.99 as is standard in the literature. We also choose a labor share of 64 percent, thus $\alpha = 0.36$. Both parameters α and β will remain unchanged for all the model specifications we consider in this paper.

We choose the same θ_w, θ_f parameters that govern the market power of firms and workers as in Huang et.al (2004) : $\theta_w = 3$, and $\theta_f = 11$. Furthermore, we pick the same CES parameter in the production function ($\nu_h = -0.7$) as in Kim and Loungani (1992) and Dhawan and Jeske (2006). We also set the household CES parameter to $\nu_h = -3.0$, which Dhawan and Jeske found to match the volatility of household energy consumption in the data very well.

Next, in Table 2 we specify six moments observed in the data that our model is supposed to replicate. We refer to Dhawan and Jeske (2006) for the details on how we computed these moments.[9] The target moments pin down six more parameters η_h, γ, η_f, δ_d, δ_k and φ. Appendix Section C has all the details.

Table 2: Targeted Moments

Moment	Value
E_h/Y	0.0456
I_D/Y	0.0932
D/Y	1.3668
E_f/Y	0.0517
K/Y	12.0000
H	0.3000

Based on Dhawan and Jeske (2006).

3.2 Monetary Policy

For the benchmark case we use the same parameters as in Leduc and Sill (2004), based on Orphanides (2001). Thus we set $\rho_r = 0.79$, $\tau_\pi^{core} = 1.80$, $\tau_\pi^e = 0.00$, and $\tau_y = 0.27$. Moreover, since we target a steady state core inflation rate of about 2 percent per year, we set $\bar{R} = 1.015$. We will also consider alternative parameterizations of this monetary policy rule in Section 4.2.

[9]Implicit in these targeted moments is the assumption that housing is part of the fixed capital stock K, not the durable goods stock D. This is because the housing stock produces housing services (both rental and owner-occupied), that are part of output. Our view therefore is that electricity and natural gas use are complementary to to the durable goods installed in residential structures (such as refrigerators, heaters, air conditioning etc.), not the housing services.

3.3 Shocks

Just as Cooley and Prescott (1995), we assume that log-TFP follows an AR(1) process:

$$z_{y,t} = \rho_z z_{y,t-1} + \varepsilon_{z,t} \tag{34}$$

where $\rho_z = 0.95$ and $\varepsilon_{z,t} \overset{iid}{\sim} N(0, \sigma_z^2)$ with $\sigma_z = 0.007$.

We assume that the energy price follows an ARMA(1,1) process:

$$p_t = \rho_p^1 p_{t-1} + \varepsilon_{p,t} + \rho_p^2 \varepsilon_{p,t-1} \text{ with } \varepsilon_{p,t} \overset{iid}{\sim} N\left(0, \sigma_p^2\right) \tag{35}$$

and use the same estimates as in Dhawan and Jeske (2006), reported in Table 3.

Table 3: ARMA(1,1) Maximum Likelihood Estimation Results

Parameter	Estimate	Standard Error
ρ_p^1	0.9753	0.0218
ρ_p^2	0.4217	0.0818
σ_p	0.0308	0.0019

From Dhawan and Jeske (2006).

3.4 Adjustment costs

Note that the model with adjustment costs generates the Neo-Keynesian Phillips Curve:

$$\pi_t = \beta E \pi_{t+1} + \frac{\theta_f - 1}{\phi_p \bar{\pi}^2} mc_t \tag{36}$$

where mc_t is marginal cost. This is the same structure as under Calvo pricing where we get:

$$\pi_t = \beta E \pi_{t+1} + \frac{(1-\upsilon)(1-\beta\upsilon)}{\upsilon} mc_t \tag{37}$$

and υ is the probability of not adjusting prices. We pick the parameters for the adjustment costs for firms and workers in order to generate the same level of rigidity as in a Calvo price setting with an average contract length of four quarters. This is the commonly used contract length in the literature.[10] We find that on the firm side:

$$\phi_p = 115.3829$$

[10]See, for example Erceg et. al. (2000).

and likewise for workers

$$\phi_w = 23.0766$$

yield nominal rigidities equivalent to Calvo price setting rules with an average contract length of four quarters.

As Dhawan and Jeske (2006) point out, without adjustment costs on durables and fixed capital investment, we generate unrealistically large investment responses to an energy price shock. Naturally, the adjustment costs cannot be calibrated from steady state moments, because they are zero in steady state. Rather, we have to use out-of-steady-state observations to estimate these parameters. Specifically, we use the same methodology as in Dhawan and Jeske (2006): we simulate the economy and set the adjustment cost parameters in order to match the second moments of the investment serios for durable and fixed investment which we found to be

We found that between 1970Q1 and 2006Q4 the volatility of HP-filtered investment were 0.0449 and 0.0526 for durables and fixed investment, respectively. We find that with

$$\begin{aligned} \phi_d &= 11.6305 \\ \phi_k &= 144.9298 \end{aligned}$$

our model subjected to the TFP and energy price shocks generates exactly those volatilities.

To get a sense of the size of these adjustment costs we simulate an economy with 200,000 quarters and compute the ratios of adjustment costs over output. We report the results in Table 4. Relative to output, the adjustment costs are small, especially for durables and fixed investment:

Table 4: Adjustment costs to output ratios (in percent)

	mean	standard deviation	95th percentile
$\frac{1}{T}\sum_{t=1}^{T} AC_t^d/Y_t$	0.0005	0.0007	0.0018
$\frac{1}{T}\sum_{t=1}^{T} AC_t^k/Y_t$	0.0009	0.0014	0.0032
$\frac{1}{T}\sum_{t=1}^{T} AC_t^w/Y_t$	0.1034	0.1463	0.3948
$\frac{1}{T}\sum_{t=1}^{T} AC_t^p/Y_t$	0.6489	0.9194	2.4842

4 Results

4.1 Benchmark

We use the stochastic perturbation method, i.e., log-linearization around the steady state, to approximate the dynamics of our economy. From the first order conditions in Appendix A, we derive 23 equations guiding the dynamic behavior of the economy. We then run the program Dynare Version 3.0 to generate a first order approximation for the policy functions (see Collard and Juillard (2001) for the methodological details).

In Figure 2, we study the effects a doubling of the energy price. Due to the ARMA(1,1) structure the peak in the energy price occurs in the second period after the shock.[11] We find that in this economy, output drops by about 4.3 percent in quarter 4. Inflation peaks at about 1.7 percent in the first period and then slowly decays over an extended period. Even after 40 quarters inflation is still at 0.7 percent above steady state. The federal funds rate jumps by about 1.0 percentage points and persistently stays above its steady state level. In fact, the interest rate is the most persistent among all series because it is not only responding to the persistent inflation shock, but also has additional persistence built in through the term $\rho_r = 0.79$ in the Taylor rule.

The reason for the persistence in inflation is that the energy price shock causes a persistent increase in marginal cost for the firm. According to equation (36), inflation is the discounted sum of future marginal costs. The higher energy price has a direct effect on marginal cost. On top of that, the higher federal funds rate increases the costs for the labor input, which accounts for 64 percent of output. Thus, higher labor cost increase the marginal cost substantially.

Finally, as pointed out in Dhawan and Jeske (2006), the investment series display a rebalancing effect, whereby the response of durables investment is stronger than the drop in fixed investment.[12] Specifically, durables investment drops sharply in the first period and fixed investment increases in the first quarter before dropping into negative territory.

4.2 Alternative monetary policy rules

Instead of solving for "the" optimal monetary policy as in Erceg et. al. (2000), we study the effect of the benchmark and four variations of the monetary policy rule to determine their relative success in cushioning the effect from the hike in energy prices and their impact on inflation. In particular, we keep the interest rate persistence parameter ρ_r constant at 0.79 and study alternative values for the parameters τ_π^{core}, τ_π^e and τ_y.[13]

Rule 1 - Benchmark: We use the benchmark Taylor rule as specified above: $\tau_\pi^{core} = 1.80$, $\tau_\pi^e = 0.00$ and $\tau_y = 0.27$ as in Leduc and Sill (2004).

[11] Also notice that the impulse responses are in terms of log deviations from steady state, thus a 100 percent increase in P^e corresponds to a log deviation of only 0.6931, not 1.0000.

[12] Notice that in Dhawan and Jeske it was the household who made both the durable and fixed investment decision. In our current economy fixed investment is done by the firm and durables investment is done by the household, so strictly speaking it is not a rebalancing of a portfolio because the two capital stocks are held by different agents. We use the phrase "rebalancing" from an economy-wide view.

[13] This is again similar to the work of Bernanke and Gertler (1999, 2001). Their work deals with asset rather than energy price shocks, but they, too, analyze the effects of changing the parameters in their generalized Taylor rule.

Rule 2 - Use headline inflation: This means we keep τ_π^{core} as in the benchmark and set:

$$\tau_\pi^e = \chi^e \tau_\pi^{core}$$

where χ^e is the steady state household energy purchases as a share of total household expenditures. In our calibration $\chi^e = 0.0556$, so we set:

$$\tau_\pi^e = 0.0556 * 1.8 = 0.1002$$

Rule 3 - Accommodate energy inflation: Notice that we can also generate interest rate rules where the central bank "accommodates" the energy price shock. Specifically, we can set the energy inflation weight to a negative number.[14] We keep $\tau_\pi^{core} = 1.80$ as in the baseline, but use a weight on the energy price changes of $\tau_\pi^e = -0.1002$.

Rule 4 - Lower weight on core inflation: We lower the coefficient on inflation. Specifically, we set $\tau_\pi^{core} = 1.50$.

Rule 5 - Higher weight on the output gap: We increase the coefficient on the output gap: $\tau_y = 0.35$.

Simulating the our economy under the alternative specifications for the Taylor rule, we did not encounter any problems with indeterminacy. This result is consistent with Carlsrom et. al. (2006) who showed, albeit in a slightly different model setup, that the Taylor principle is robust to using different definitions of inflation, namely core CPI or headline CPI. What is noteworthy, though, is that even with a *negative* weight on the energy portion of inflation, as under Rule 3, our equilibrium is still determinate.

Table 5 summarizes the alternative monetary policy rules we study. We plot the impulse response functions for output, inflation and the federal funds rate under the benchmark and the four alternative monetary policy rules in Figure 3.

We find that a central bank that uses headline inflation in the Taylor rule (Rule 2) causes a large drop in output, almost 9 percent in the first quarter. Over the whole transition, output is the lowest among the five policy rules we consider. In the first period, inflation is indeed slightly lower than in the benchmark but then it stays persistently above the benchmark level. The federal funds rate is above that in the under the Benchmark Taylor rule.

If the central bank accommodates the energy price shock through a negative weight on energy

[14]Notice that π_t^e is the gross increase in the energy price relative to the core basket of goods, not the nominal gross increase in energy prices which would be $\pi_t^e \cdot \pi_t$. In the remainder of the paper when we refer to energy price inflation, we mean the change in the energy price relative to the core goods, i.e., π_t^e.

Table 5: Alternative Interest Rate Rules

		τ_π^{core}	τ_π^{e}	τ_y
Rule 1	Benchmark	1.8000	0.0000	0.2700
Rule 2	Use headline inflation	1.8000	0.1002	0.2700
Rule 3	Accommodate energy inflation	1.8000	−0.1002	0.2700
Rule 4	Lower weight on core inflation	1.5000	0.0000	0.2700
Rule 5	Higher weight on the output gap	1.8000	0.0000	0.3500

Note: We keep the persistence parameter ρ_r at the baseline level of 0.79 for all alternative policy rules.

price inflation (Rule 3), output stays above the steady state level for two quarters before dropping into negative territory.[15] Along the transition path back to the steady state, output is consistently above that of the benchmark Taylor rule. Core inflation spikes at about 2 percent above steady state, though for one period only. After that, inflation is the lowest among the five policy rules. With regards to monetary policy, despite higher core inflation in the first period, the federal funds rate barely increases due to the negative weight on energy inflation. After that both the interest rate persistence and the low core inflation keep the federal funds rate the lowest among the five policy rules.

Finally, we find that putting a low weight on core inflation (Rule 4) or a high weight on the output gap (Rule 5) cushions the output drop. In fact, in both cases output even increases slightly in the first quarter. But these two rules also cause higher core inflation and federal funds rates.

We conclude that in this economy using headline inflation in the Taylor rule is inferior both in terms of the output loss and core inflation. Altering the coefficients on core inflation and the output gap poses a tradeoff between inflation and output. Rule 3, which accommodates energy price shocks through a negative weight on the energy inflation component, is therefore unique in that it cushions both inflation and the output loss. In other words, the central bank can accommodate the energy price shock without trading off higher inflation for it.

How is it possible that the central bank can get something for nothing? As Dhawan and Jeske (2006) point out, the rebalancing of durable and fixed capital plays a key role in the response to energy price shocks. We can show that different monetary policy rules have different effects on this rebalancing. In Figure 4 we plot the two investment series under three alternative specifications for the Taylor rule: The benchmark (Rule 1), using headline inflation (Rule 2) and Accommodating energy inflation (Rule 3). We find that the Taylor rule with headline inflation discourages rebalancing. Both IRFs are lower than under the benchmark but the effect is much

[15]This outcome is similar to that in the experiment in Carlstrom and Fuerst (2006a), who show that if the central bank accommodates the energy price shock for four quarters, output actually increases temporarily.

stronger for fixed investment. Going from Rule 1 to Rule 2 causes a drop in the fixed investment IRF by about 40 percentage points in the first quarter, while the effect is only about 15 percentage points for durables investment.

In contrast, going from Rule 1 to Rule 3 encourages rebalancing. Fixed investment is much higher than under the benchmark Taylor rule. Intriguingly, even durables investment drops less than under the benchmark Taylor rule. This is clearly due to the the income effect because output is a lot higher under Rule 3.

Why does Rule 3 encourage rebalancing in favor of fixed investment and ultimately a lower output loss? We take a closer look at the impulse response functions under Rule 3. There are multiple channels at work. First, the spike in core inflation, which is more pronounced under Rule 3 than under Rules 1 and 2, causes a more negative wealth effect for households who hold the nominal money balance at the beginning of the period. Because of the cash in advance constraint, households have to finance their consumption of N, E_h and I_d out of the lower real money balance, while at the same time facing higher energy prices. Also notice that while deposits pay an above steady state interest rate, the federal funds rate under Taylor rule 3 is still far below those under Rules 1 and 2, see Figure 3 lower panel. Faced with a negative wealth effect from both high inflation and relatively low deposit rates, households are encouraged to reduce their durable goods consumption.

A side effect of the negative wealth effect is that households work more during the initial two quarters. See Figure 5 where we plot the response to the doubling in the energy price in hours and the wage. Under Rule 3 we observe the largest increase in hours in the initial periods. The drop in the real wage W in the first quarter is also more pronounced under Rule 3. This is because of the spike in inflation. Households have a harder time time passing on the core price increase into higher wages because of the nominal rigidity. After the initial period however, the wage under Rule 3, while below steady state, stays above that under Rules 1 and 2. This is clearly due to smaller drop in the fixed capital stock along the transition path which makes labor more productive.

For firms the increase in the energy price has a negative effect, but under Rule 3 it is cushioned substantially. The large increase in hours coupled with lower wages makes capital more productive. Indeed the increase in labor input more than makes up for the decline in firm energy use to cause an initial spike in fixed investment. A higher federal funds rate means higher costs to finance the wage bill, but notice that the federal funds rate increases only mildly in the first two quarters, only about a quarter percentage point and thus much less than under the other Taylor rules. The lower wage more than compensates for that.

4.3 Robustness check

In this section we study how our results change under different specifications of the basic model structure. Specifically, we perform robustness analysis with respect to the parameters outside of the Taylor rule, for example the persistence of the energy price shock or the degree of nominal rigidities. We consider a total of seven calibrations (including the benchmark calibration) and in each case study the five different Taylor rule parameterizations outlined in Subsection 4.2. Thus, we solve for impulse response functions in a total of 35 different economies and in each case compare both the cumulative output loss and the impact on inflation.

As a measure of the output loss we use:

$$L^y = \frac{\sum_{t=1}^{\infty} \beta^{t-1} \left(\exp\left(\tilde{y}_t\right) - 1\right)}{\sum_{t=1}^{\infty} \beta^{t-1}} = (1-\beta) \sum_{t=1}^{\infty} \beta^{t-1} \left(\exp\left(\tilde{y}_t\right) - 1\right) \tag{38}$$

where $\tilde{y}_t$ is the impulse response function, i.e., the log deviation from the steady state. One can think of L^y as translating the time-varying output loss in the impulse response function into one constant permanent loss in every period. We present the results in Table 6, where the columns correspond to the alternative monetary policy rules. The first column is the benchmark Taylor rule and the four additional columns are for the alternative parameterizations, as discussed in Subsection 4.2. The rows correspond to alternative calibrations. The first row is for the benchmark model calibration, as detailed in Section 3 and the other rows are for the five alternative calibrations discussed below.

We also compare the inflation path across the different calibrations and Taylor rules. Our measure of the total impact on inflation is:

$$PL = \exp\left(\sum_{t=1}^{\infty} \tilde{\pi}_t\right), \tag{39}$$

where $\tilde{\pi}_t$ is the inflation log-deviation from its steady state. The term PL is the total change in the price level due to the energy price shock and subsequent monetary policy response. In other words, $1 + PL$ is the factor by which we multiply the exponentially growing price level in an economy without an energy price shock. We report these results in Table 7.

Since our main focus will be how accommodation of energy prices affects macroeconomic outcomes, we also report the success of Rule 3 in reducing the output drop and the inflation impact relative to the benchmark Taylor rule 1. Specifically, Table 8 reports how much lower the output drop and inflation impact are (in percent) if the central bank uses Rule 3 rather than Rule 1.

As far as the benchmark calibration (row 1) is concerned, results on the cumulative output loss are consistent with the observations from the impulse responses; using headline inflation (Rule 2) causes a larger output loss than in the benchmark, whereas accommodating the energy price shock (Rule 3) generates the lowest output loss, almost 12 percent below that under Rule 1. The output drop is lower than under the benchmark Taylor rule if the central bank is soft on core inflation (Rule 4) or puts a large weight on the output gap (Rule 5).

Clearly, the impact of energy price shocks is substantial. In the benchmark economy (row 1) with the benchmark Taylor rule (column 1), the price level will eventually be be about 73.76 percent larger than that in an economy without an energy price shock. Notice that as in the case of the output drop, when comparing Rules 1, 2 and 3, the rules with lower weights on energy price inflation do better than those with higher weights. Most importantly, under Rule 3 the impact on the price level is about 11 percent lower than under benchmark Taylor rule. Rules 4 and 5 cause a larger increase in the price level.

The results in the three tables about the benchmark calibration (row 1) are thus consistent with the impulse response functions in figure 3: Comparing rules 1 through 3 we find that the lower the weight on energy inflation, the lower the impact on both inflation and the output drop. Comparing Rule 1 with Rules 4 and 5 we find that there is a tradeoff between output and inflation. Specifically, both rules 4 and 5 are able to cushion the output drop but at the cost of higher inflation.

To see how robust our results are we study the output drop and the permanent effect on the price level in economies where we change parameters other than those in the Taylor rule. In each case we recalibrate the adjustment cost parameters ϕ_d and ϕ_k to match the observed second moments of the investment series.[16]

Simple household problem - no durables and no household energy use: First, we replicate the result from Dhawan and Jeske (2006) that durable goods and household energy use matter: The output drop in the economy with the simple household problem is larger than under the benchmark where we explicitly model household energy use and durable goods. Moreover, comparing Rules 1, 2 and 3, the cumulative output drop is roughly the same whether the central bank uses core CPI, headline CPI or a negative weight on energy inflation. This is in line with Figure 4, where we learned that different monetary rules have different effects on the rebalancing between durables and fixed capital investment. Shutting down this rebalancing channel, we lose the differential impact of the alternative Taylor rules in columns 2 and 3.

No wage rigidity: Without wage rigidity the cumulative impact on output is larger than under the benchmark calibration. The ordering in output losses and inflation impact between

[16]One exception is an economy without durable goods where we match the volatility of fixed investment only.

rules 1 through 3 is unchanged, though. As before, Rule 2 performs the worst among these three rules and Rule 3 performs the best. What does change between the benchmark calibration and the economy without wage rigidity is that now the output drop under rules 4 and 5 is larger than under the benchmark Taylor rule. There is no tradeoff any more: Being easy on core inflation (Rule 4) of the output gap (Rule 5) exacerbates the output drop and causes more inflation. This is the same result as in Leduc and Sill (2004) who showed that lower weights on core inflation or higher weights on the output gap cause larger output drops and higher inflation than in the benchmark Taylor rule.

No price rigidity: Qualitatively the results are the same as in the case of no wage rigidity. The ordering between rules 1 through 3 is the same as before and rules 4 and 5 cause larger output drops and inflation.

No wage, no price rigidity: Without any nominal rigidities, i.e., with zero adjustment costs for both wages and prices we maintain the ordering between Rules 1 through 3 for both output drop and inflation.

We also find that rules 4 and 5 are most damaging in the economy without nominal rigidities. Notice that this economy is most similar to that of Leduc and Sill (2004). Their nominal rigidities were essentially equal to zero. For example their price adjustment cost parameter ϕ_p corresponded to Calvo contracts with a length of 1.02 quarters, or alternatively, 98 percent of all firms change prices every quarter. In our economy, therefore, we replicate their result that rules 4 and 5 are inferior with respect to both output and inflation.

Different firm markups: We also check how different values for θ_f change our results. The benchmark value of $\theta_f = 11$ generates a firm markup of $\mu_p = \frac{1}{\theta_f - 1} = 0.10$. Other studies found the markup to be in the range $\mu \in [0.05, 0.20]$.[17] We thus simulate the economy for $\theta_f = 6$ and $\theta_f = 21$. The results are again consistent with those in the benchmark. Most importantly, headline inflation in the Taylor rule (Rule 2) causes a larger output drop and more inflation and accommodating energy inflation (Rule 3) causes a lower output drop and less inflation than in the benchmark.

Our robustness analysis shows that along a wide variety of alternative calibrations, the lower the weight on energy inflation in the Taylor rule the lower is the output drop and the inflation impact. Specifically, using headline inflation, which implies a positive weight on energy price inflation, exacerbates the output loss and inflation impact relative to the benchmark Rule 1, while

[17] See Huang et. al. (2004). Christiano et. al. (2005) use a markup of 0.20. Basu and Fernald (2000) find a markup of 0.05.

Table 6: Cumulative Output Loss (in percent) under different modeling assumptions and monetary policy rules.

	Monetary Policy				
	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5
benchmark calibration	1.4922	1.6597	1.3141	1.3742	1.4404
no D, E_h	1.5766	1.5783	1.5750	1.4484	1.5251
no wage rigidity	1.9144	2.1079	1.7139	2.2592	2.1050
no price rigidity	1.7428	1.8974	1.5852	1.8119	1.7885
no wage, no price rigidity	2.2191	2.3172	2.1208	3.4444	2.7250
$\theta_f = 6$	1.6448	1.8017	1.4797	1.5265	1.5952
$\theta_f = 21$	1.4128	1.5861	1.2275	1.2966	1.3607

Table 7: Permanent change in the price level as a multiple of an economy without an energy price shock

	Monetary Policy				
	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5
benchmark calibration	0.7376	0.8195	0.6557	1.0602	0.9193
no D, E_h	0.7865	0.7873	0.7857	1.1265	0.9803
no wage rigidity	1.0025	1.1099	0.8952	1.8741	1.4319
no price rigidity	0.8778	0.9691	0.7865	1.4098	1.1606
no wage, no price rigidity	1.2246	1.3189	1.1304	3.0998	1.9800
$\theta_f = 6$	0.8269	0.9065	0.7473	1.1943	1.0337
$\theta_f = 21$	0.6930	0.7762	0.6097	0.9941	0.8626

Note: A value of 0.8195 in row 1, column 2 means that in the benchmark calibration with the Taylor rule 2 (with headline inflation), a doubling of the energy price drives the core price level 81.95 percent higher in the long term.

Rule 3, which accommodates energy price inflation, cushions the drop and the price increase. This result seems to come mainly from the rebalancing effect, because the only case in which the choice of the inflation measure in the Taylor rule does not matter much, is when the model lacks the choice between durable and fixed investment.

We also confirm Leduc and Sill's (2004) result that with low levels of nominal rigidities the central bank exacerbates the output loss following an energy price hike by choosing a low weight on core inflation or a high weight on the output gap. However, we show that this result is not all that robust. For higher levels of nominal rigidities we can reverse this finding. Specifically, if we use adjustment cost parameters that correspond to commonly used degrees of Calvo-type price stickiness – price and wage adjustments are done on average only every four quarters – we find that a central bank accommodating an energy price hike can cushion the output drop, though at the cost of a higher increase in the price level.

Table 8: Percentage Reduction of the output loss and price level impact if the central bank accommodates energy inflation (Rule 3) relative to benchmark Taylor rule

	Output drop	Impact on price level
benchmark calibration	11.93	11.11
no D, E_h	0.11	0.11
no wage rigidity	10.47	10.71
no price rigidity	9.04	10.40
no wage, no price rigidity	4.43	7.70
$\theta_f = 6$	10.03	9.63
$\theta_f = 21$	13.11	12.01

5 Conclusion

Which inflation measure should the central bank focus on in its Taylor rule, core or headline? To answer this question we clearly need a model with energy price shocks, since a large part of the difference between the two measures comes from the volatile energy price series. We set up a model with money, durable goods, nominal rigidities and energy price shocks to study how the economy behaves under different monetary policy rules after being subjected to an energy price hike. Specifically, we allow a generalized functional form for the Taylor rule that includes a term for energy price inflation in addition to core inflation and the output gap. Clearly, a central bank using headline inflation is a special case of this rule.

A negative weight on energy price inflation, which we view as accommodating the energy price shock, cushions the output drop while actually allowing a lower price increase than under the benchmark Taylor rule. We conclude that a central bank can in fact accommodate an energy price shock, as long as the accommodation refers to energy prices only, while still being vigilant on core inflation. Conversely, using headline inflation in a Taylor rule appears to be a bad idea, both in terms of the output drop and the inflation impact. This result is robust along a wide variety of alternative calibrations. Only in the absence of household durable goods would the weight on energy price inflation be irrelevant. This indicates that the rebalancing between durable and fixed capital investment plays the key role in explaining the differential impacts of monetary policy. Thus, we found a new application of the rebalancing effect of Dhawan and Jeske (2006), namely in the transmission of monetary policy following an energy price shock.

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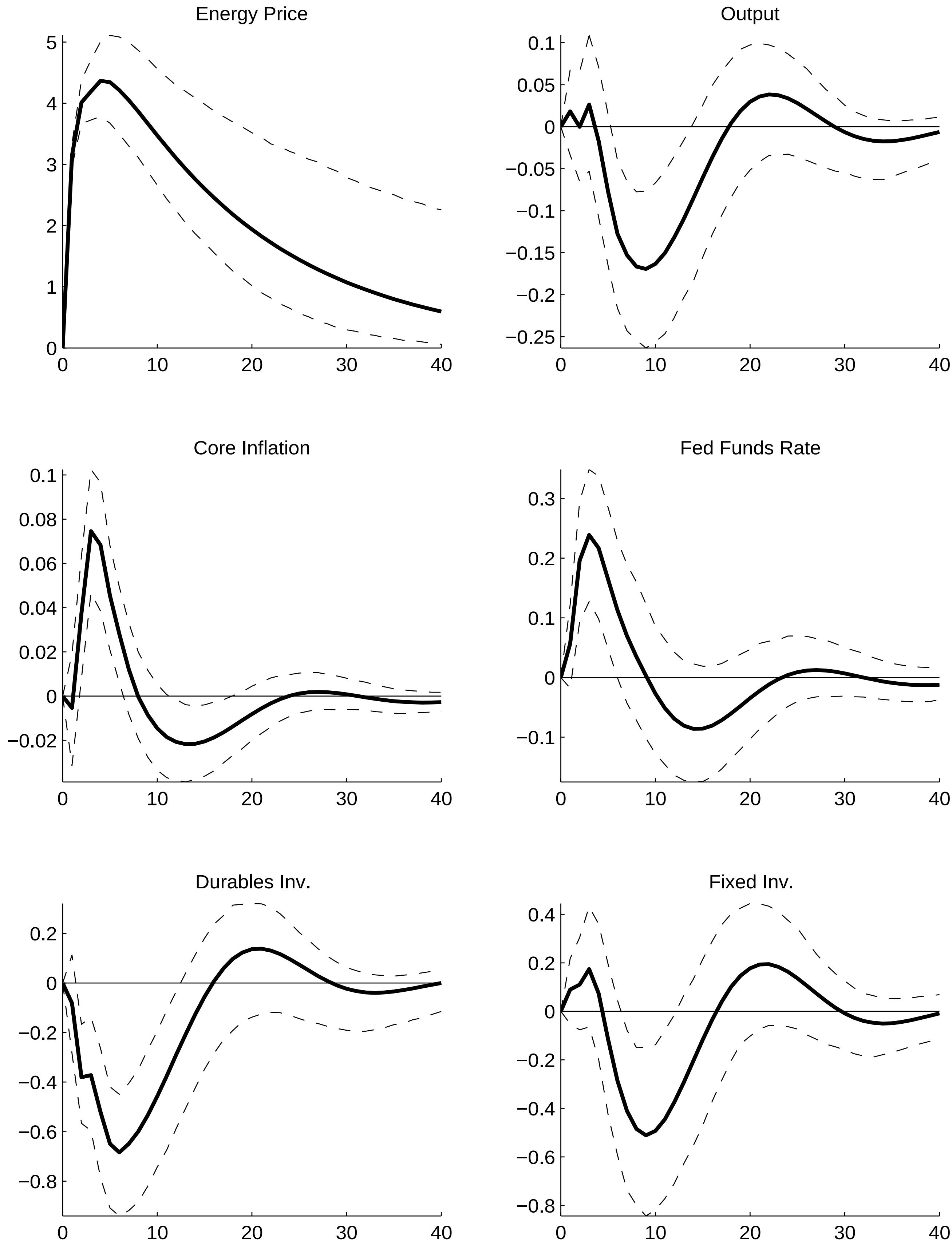
Figure 1: VAR impulse responses to an energy price shock (in percent)



Note: Data cover quarters 1970:1-2006:4. The dashed lines are the 68 percent Sims and Zha (1999) error bands.

Figure 2: Impulse Response Functions to a doubling of the energy price shock in the benchmark

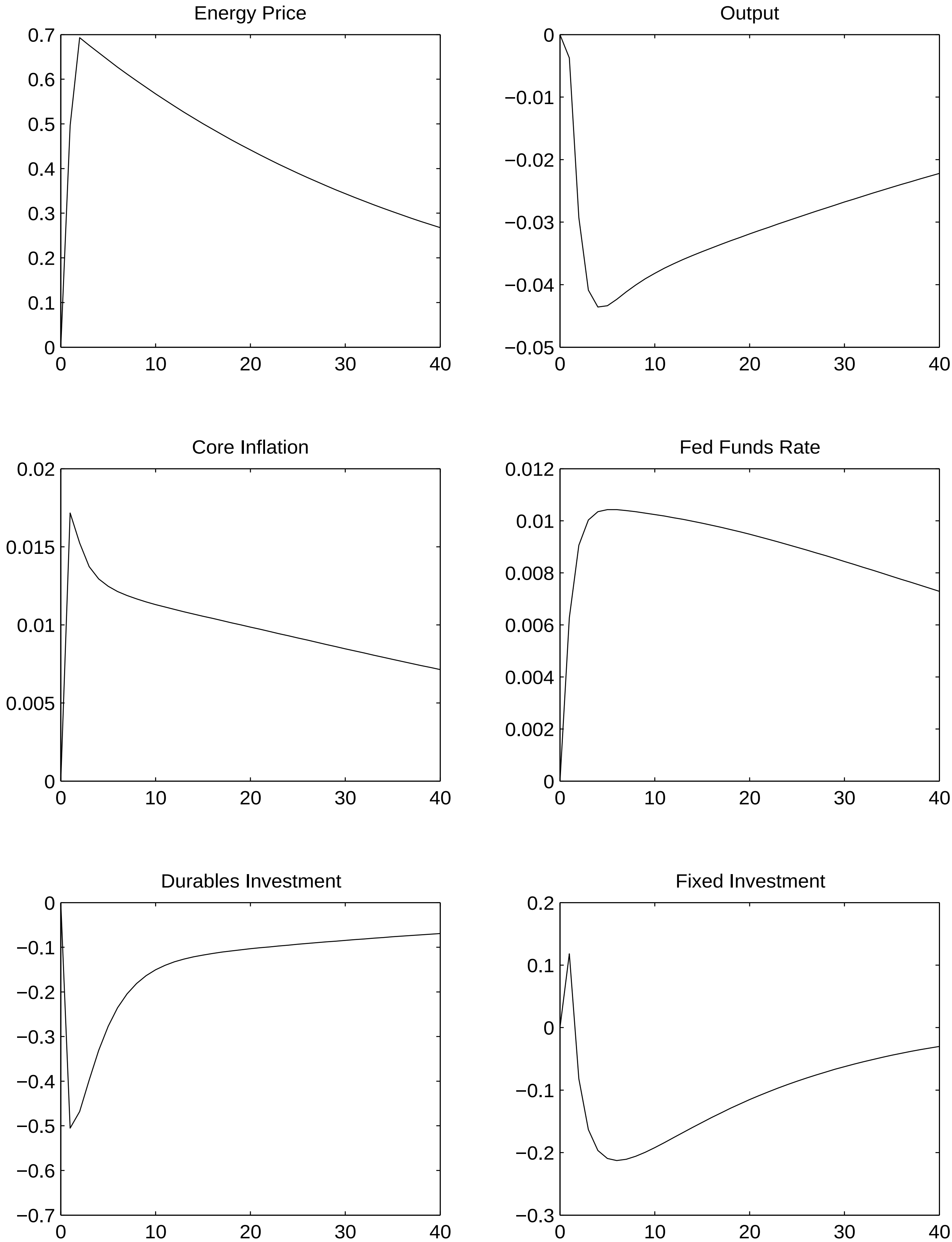


Figure 3: Model impulse responses to a doubling in the energy price: Alternative Policy Rules

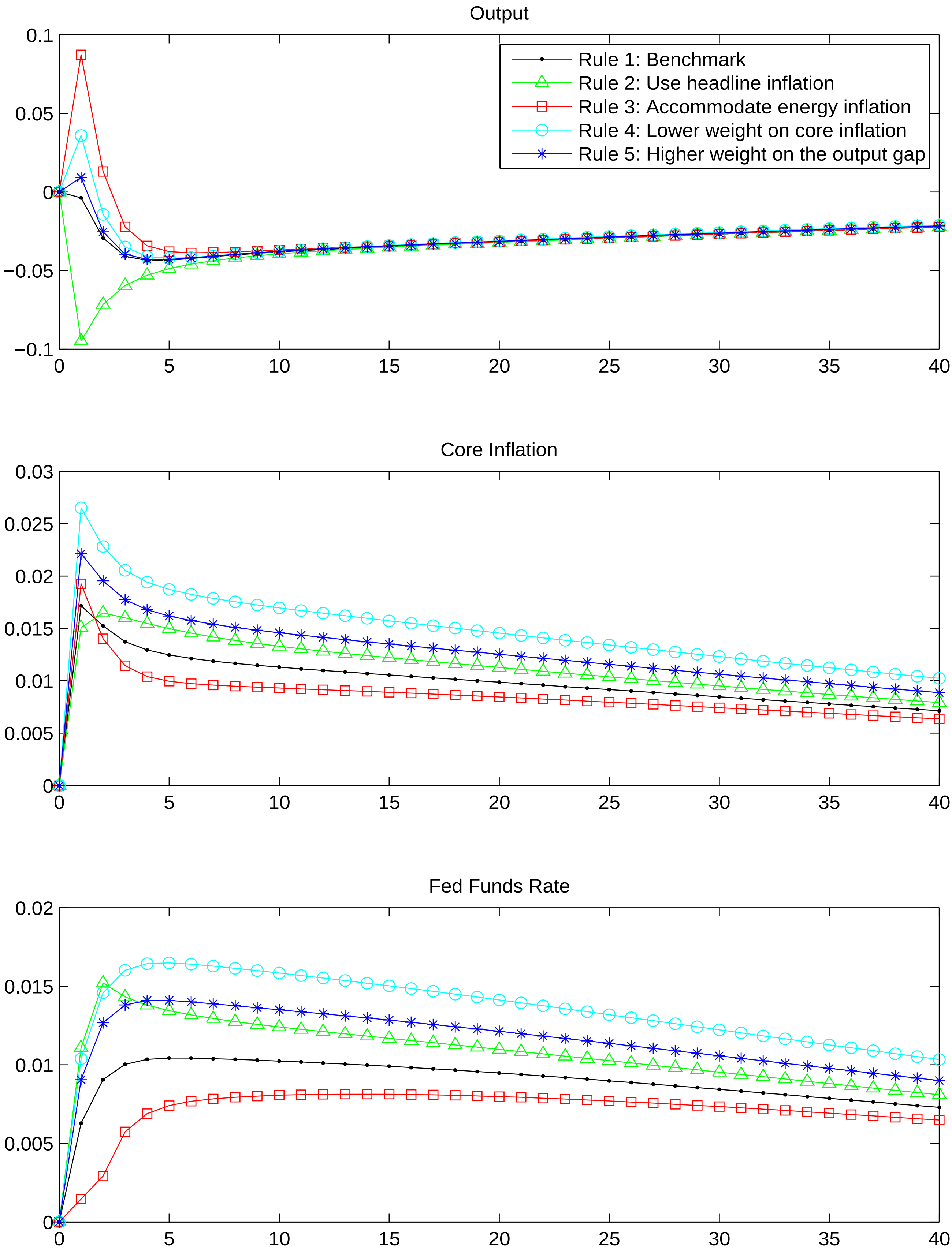
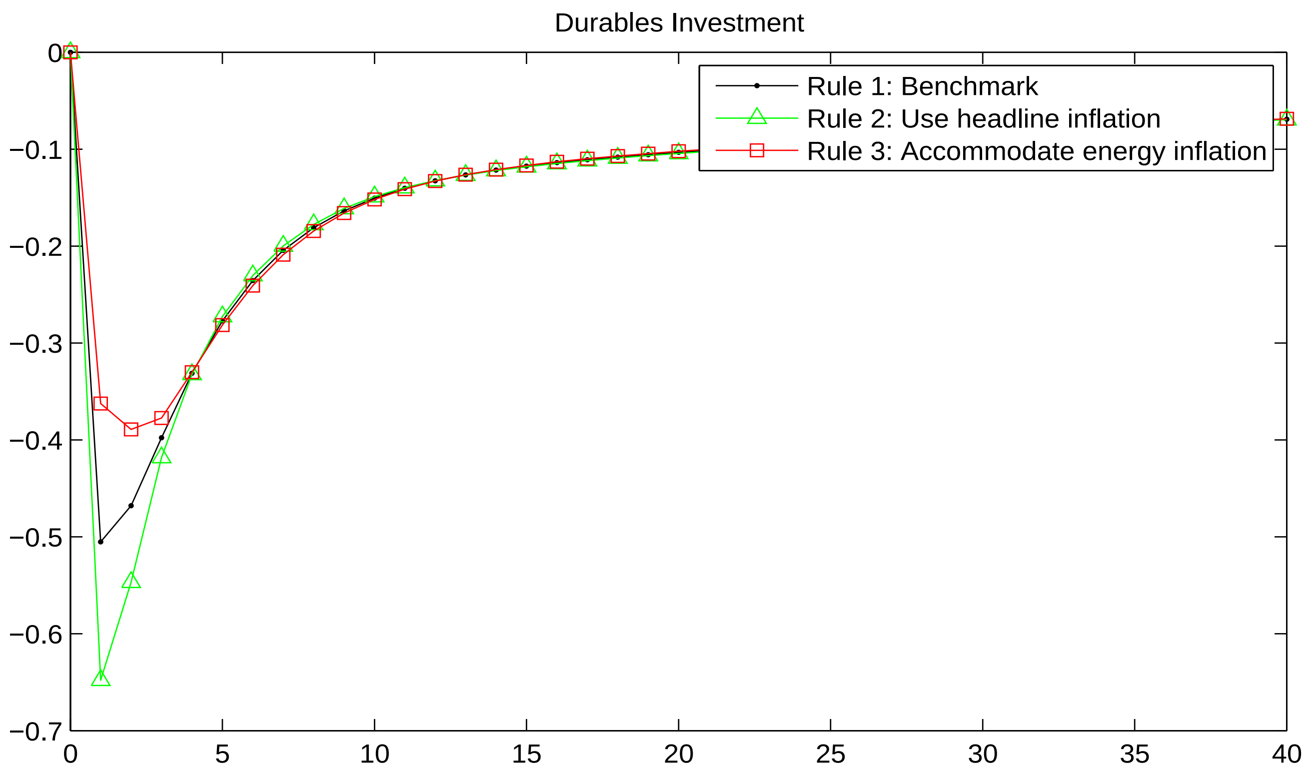


Figure 4: Model impulse responses to a doubling in the energy price: Investment Series under Alternative Policy Rules



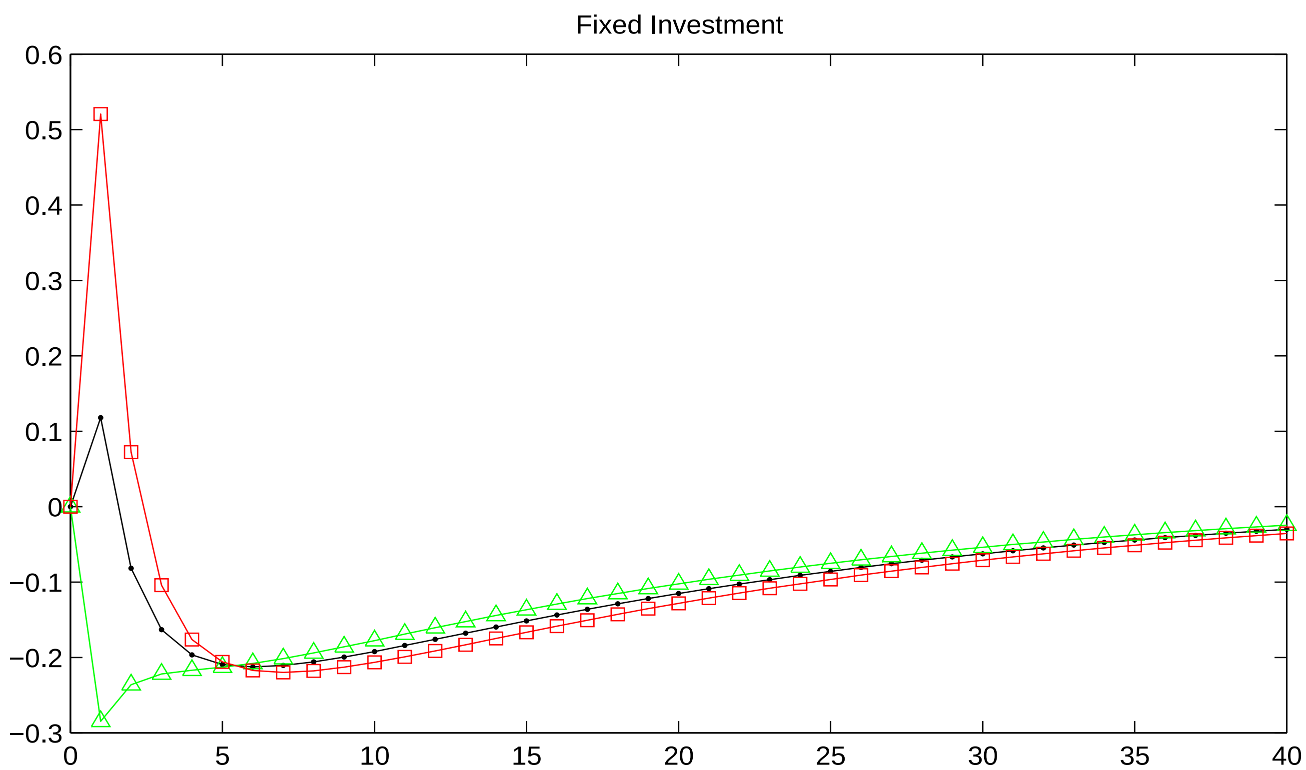
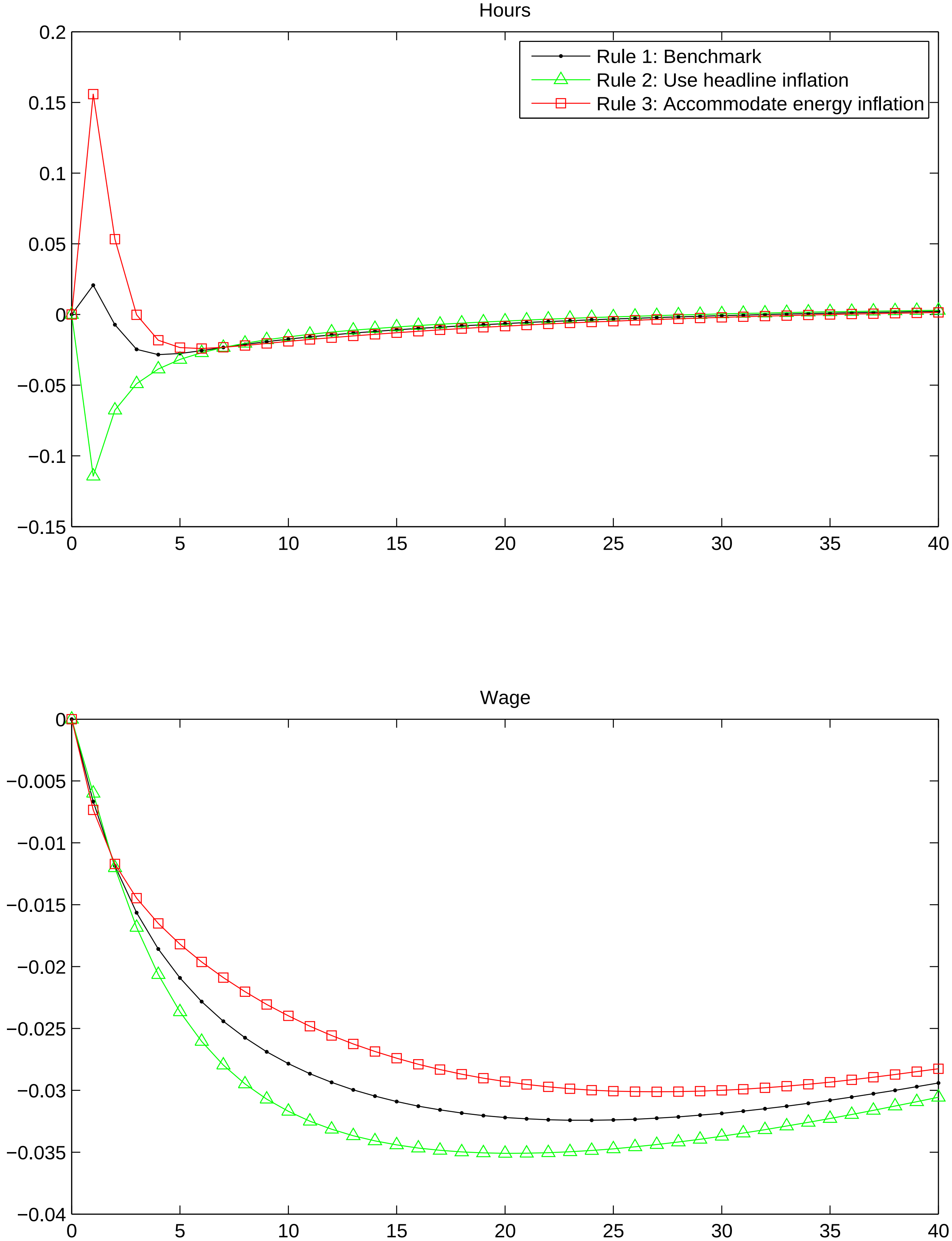


Figure 5: Model impulse responses to a doubling in the energy price: Alternative Policy Rules



Appendix

A First order conditions:

A.1 Consumer

In the first order conditions we skip the expectation operator when referring to $t+1$ variables to save on notation. Assume the cash in advance constraint binds. Then the Lagrangian for the consumer is

$$\begin{aligned} L^c &= E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_{i,t}^A, H_{i,t}\right) \\ &+ \sum_{t=0}^{\infty} \beta^t \lambda_{i,t}^B \left[-AC_{i,t}^w - AC_{i,t}^d + R_t \frac{DP_{i,t}}{P_t} + H_{i,t}^{1-1/\theta_w} H_t^{1/\theta_w} W_t + \Pi_{i,t}^f + \frac{\Pi_{i,t}^b}{P_t} - \frac{M_{i,t}}{P_t}\right] \\ &+ \sum_{t=0}^{\infty} \beta^t \lambda_{i,t}^C \left[\frac{M_{i,t-1}}{P_t} - \frac{DP_{i,t}}{P_t} - N_t - I_{d,i,t} - P_t^e E_{h,i,t}\right] \\ &+ \sum_{t=0}^{\infty} \beta^t \lambda_{i,t}^I \left[I_{d,i,t} + (1-\delta_d) D_{i,t-1} - D_{i,t}\right] \end{aligned}$$

- Nondurables $N_{i,t}$:

$$\beta^t u_1\left(C_{i,t}^A, H_{i,t}\right) \frac{\partial C_{i,t}^A}{\partial N_{i,t}} = \beta^t \lambda_{i,t}^C \tag{A-1}$$

- Durables Stock $D_{i,t}$:

$$\begin{aligned} 0 &= \beta^{t+1} u_1\left(C_{i,t+1}^A, H_{i,t+1}\right) \frac{\partial C_{i,t+1}^A}{\partial D_{i,t}} \\ & -\beta^{t+1} \lambda_{i,t+1}^B \frac{\partial AC_{i,t+1}^d\left(I_{d,i,t+1}, D_{i,t}\right)}{\partial D_{i,t}} - \beta^t \lambda_{i,t}^I + (1-\delta_d) \beta^{t+1} \lambda_{i,t+1}^I \end{aligned} \tag{A-2}$$

- Energy $E_{h,i,t}$:

$$\beta^t u_1\left(C_{i,t}^A, H_{i,t}\right) \frac{\partial C_{i,t}^A}{\partial E_{h,i,t}} = \beta^t P_t^e \lambda_{i,t}^C \tag{A-3}$$

- Investment in Durables $I_{d,i,t}$:

$$\beta^t \lambda_{i,t}^B \frac{\partial AC_{i,t}^d\left(I_{d,i,t}, D_{i,t-1}\right)}{\partial I_{d,i,t}} + \beta^t \lambda_{i,t}^C = \beta^t \lambda_{i,t}^I \tag{A-4}$$

- Money holdings M_t :

$$\frac{\beta^t \lambda_{i,t}^B}{P_t} = \frac{\beta^{t+1} \lambda_{i,t+1}^C}{P_{t+1}} \tag{A-5}$$

- Deposits:

$$\frac{\beta^t \lambda_{i,t}^B}{P_t} R_t = \frac{\beta^t \lambda_{i,t}^C}{P_t} \tag{A-6}$$

- Hours $H_{i,t}$:

$$\begin{aligned} 0 &= \beta^t u_2\left(C^A_{i,t}, H_{i,t}\right) + \beta^t \lambda^B_{i,t}\left(1 - \frac{1}{\theta_w}\right) W_t \left(\frac{H_t}{H_{i,t}}\right)^{1/\theta_w} \\ & -\beta^t \lambda^B_{i,t} \frac{\partial AC^w_{i,t}}{\partial W_{i,t}} \frac{\partial W_{i,t}}{\partial H_{i,t}} - \beta^{t+1} \lambda^B_{i,t+1} \frac{\partial AC^w_{i,t+1}}{\partial W_{i,t}} \frac{\partial W_{i,t}}{\partial H_{i,t}} \end{aligned} \tag{A-7}$$

Thus:

$$\begin{aligned} -\beta^t u_2\left(C^A_{i,t}, H_{i,t}\right) &= \beta^t \lambda^B_{i,t}\left(1 - \frac{1}{\theta_w}\right) W_t \left(\frac{H_t}{H_{i,t}}\right)^{1/\theta_w} \\ & -\frac{\partial W_{i,t}}{\partial H_{i,t}} \left[\beta^t \lambda^B_{i,t} \frac{\partial AC^w_{i,t}}{\partial W_{i,t}} + \beta^{t+1} \lambda^B_{i,t+1} \frac{\partial AC^w_{i,t+1}}{\partial W_{i,t}}\right] \end{aligned} \tag{A-8}$$

Notice that

$$\frac{\partial W_{i,t}}{\partial H_{i,t}} = -\frac{1}{\theta_w} H_{i,t}^{-1/\theta_w - 1} W_t H_t^{1/\theta_w} \tag{A-9}$$

- Plus the constraints and the definition of C^A.

We can assume that in a symmetric equilibrium $W_{i,t} = W_t, H_{i,t} = H_t, N_{i,t} = N_t$ and so on. Also, we eliminate multiplier λ^C :

$$\lambda^C_t = \lambda^B_t R_t \tag{A-10}$$

Then the Consumer First Order Conditions become

- Non-Durables:

$$u_1\left(C^A_t, H_t\right) \frac{\partial C^A_t}{\partial N_t} = \lambda^B_{i,t} R_t \tag{A-11}$$

- Durables:

$$0 = \beta u_1\left(C^A_{t+1}, H_{t+1}\right) \frac{\partial C^A_{t+1}}{\partial D_t} - \beta \lambda^B_{t+1} \frac{\partial AC^d_{t+1}\left(I_{d,t+1}, D_t\right)}{\partial D_t} - \lambda^I_t + (1 - \delta_d) \beta \lambda^I_{t+1} \tag{A-12}$$

- Energy:

$$u_1\left(C^A_t, H_t\right) \frac{\partial C^A_t}{\partial E_{h,t}} = P^e_t R_t \lambda^B_{i,t} \tag{A-13}$$

- Investment:

$$\lambda^B_t \left[\frac{\partial AC^d_t\left(I_{d,t}, D_{t-1}\right)}{\partial I_{d,t}} + R_t\right] = \lambda^I_t \tag{A-14}$$

- Money holdings and deposits

$$1 = \beta \frac{P_t}{P_{t+1}} \frac{\lambda^B_{t+1}}{\lambda^B_{i,t}} R_{t+1} \tag{A-15}$$

- Hours:

$$\begin{aligned} -\beta^t u_2\left(C^A_t, H_t\right) &= \beta^t \lambda^B_t \left(1 - \frac{1}{\theta_w}\right) W_t \\ & + \frac{1}{\theta_w} \frac{W_t}{H_t} \left[\beta^t \lambda^B_{i,t} \frac{\partial AC^w_{i,t}}{\partial W_{i,t}} + \beta^{t+1} \lambda^B_{i,t+1} \frac{\partial AC^w_{i,t+1}}{\partial W_{i,t}}\right] \end{aligned} \tag{A-16}$$

- Budget Constraint

$$M_t = -P_t \left(AC_t^w + AC_t^d \right) + R_t DP_t + P_t W_t H_t + P_t \Pi_t^f + \Pi_t^b \tag{A-17}$$

- Cash in advance:

$$N_t + I_{d,t} + P_t^e E_{h,t} = \frac{M_{t-1}}{P_t} - \frac{DP_t}{P_t} \tag{A-18}$$

- Definition of durables investment:

$$D_t = I_{d,t} + (1 - \delta_d) D_{t-1} \tag{A-19}$$

- Consumption aggregator:

$$C_t^A = N_t^{1-\gamma} \left(\eta_h D_{t-1}^{\nu_h} + (1 - \eta_h) E_{h,t}^{\nu_h} \right)^{\frac{\gamma}{\nu_h}} \tag{A-20}$$

A.2 Firms

The derivative of revenue $Y_{j,t} Y_t^{1/\theta_f}$ with respect to hours, capital and energy is:

$$\frac{\partial}{\partial H_{j,t}} Y_{j,t}^{1-1/\theta_f} Y_t^{1/\theta_f} = (1 - 1/\theta_f) \left(\frac{Y_t}{Y_{j,t}} \right)^{1/\theta_f} MPL_{j,t} \tag{A-21}$$

$$\frac{\partial}{\partial K_{j,t}} Y_{j,t}^{1-1/\theta_f} Y_t^{1/\theta_f} = (1 - 1/\theta_f) \left(\frac{Y_t}{Y_{j,t}} \right)^{1/\theta_f} MPK_{j,t} \tag{A-22}$$

$$\frac{\partial}{\partial E_{f,j,t}} Y_{j,t}^{1-1/\theta_f} Y_t^{1/\theta_f} = (1 - 1/\theta_f) \left(\frac{Y_t}{Y_{j,t}} \right)^{1/\theta_f} MPE_{j,t} \tag{A-23}$$

The firm's Lagrangian

$$\begin{aligned} L^f &= E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^B \left[Y_{j,t}^{1-1/\theta_f} Y_t^{1/\theta_f} - W_t H_{j,t} R_t - I_{k,j,t} - P_t^e E_{f,j,t} - AC_t^k - AC_{j,t}^p \right] \\ &+ \sum_{t=0}^{\infty} \beta^{t+1} \lambda_{j,t}^F \left[(1 - \delta_k) K_{j,t-1} + I_{k,j,t} - K_{j,t} \right] \end{aligned} \tag{A-24}$$

First order conditions:

- Hours:

$$(1 - 1/\theta_f) \left(\frac{Y_t}{Y_{j,t}} \right)^{1/\theta_f} MPL_{j,t} = W_t R_t + \frac{\partial AC_{j,t}^p}{\partial H_{j,t}} + \beta \frac{\lambda_{t+1}^B}{\lambda_t^B} \frac{\partial AC_{j,t+1}^p}{\partial H_{j,t}} \tag{A-25}$$

- Capital:

$$\begin{aligned} \beta^t \lambda_t^F &= \beta^{t+1} \lambda_{t+1}^B MPK_{j,t+1} \left[(1 - 1/\theta_f) \left(\frac{Y_{t+1}}{Y_{j,t+1}} \right)^{1/\theta_f} \right] \\ &\quad - \beta^{t+1} \lambda_{t+1}^B \frac{\partial AC_{j,t+1}^p}{\partial K_{j,t}} - \beta^{t+2} \lambda_{t+2}^B \frac{\partial AC_{j,t+2}^p}{\partial K_{j,t}} \\ &\quad - \beta^{t+1} \lambda_{t+1}^B \frac{\partial AC_{t+1}^k}{\partial K_t} + \beta^{t+1} \lambda_{j,t+1}^F (1 - \delta_k) \end{aligned} \tag{A-26}$$

- Energy:

$$\left(1-1/\theta_f\right)\left(\frac{Y_t}{Y_{j,t}}\right)^{1/\theta_f} MPE_{j,t} = P_t^e + \frac{\partial AC_{j,t}^p}{\partial E_{f,j,t}} + \beta\frac{\lambda_{t+1}^B}{\lambda_t^B}\frac{\partial AC_{j,t+1}^p}{\partial E_{f,j,t}} \tag{A-27}$$

- Investment:

$$\beta^t\lambda_t^B\left[1+\frac{\partial AC_t^k}{\partial I_{k,j,t}}\right] = \beta^t\lambda_{j,t}^F \tag{A-28}$$

- Definition of investment:

$$K_{j,t} = \left(1-\delta_k\right)K_{j,t-1} + I_{k,j,t} \tag{A-29}$$

- Definition of profits:

$$\Pi_{j,t} = Y_{j,t}Y_t^{1/\theta_f} - W_tH_{j,t}R_t - I_{k,j,t} - P_t^eE_{f,j,t} - AC_t^k - AC_{j,t}^p \tag{A-30}$$

Again, we can simplify everything by noting that we are in a symmetric equilibrium:

- Hours:

$$\begin{aligned} W_tR_t &= \left[1-1/\theta_f + \frac{\phi_p}{\theta_f}\left(\pi_t-\bar{\pi}\right)\pi_t - \frac{\phi_p}{2}\left(\pi_t-\bar{\pi}\right)^2\right]MPL_t \\ &\quad -\beta\frac{\lambda_{t+1}^B}{\lambda_t^B}\frac{\phi_p}{\theta_f}\left(\pi_{t+1}-\bar{\pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}MPL_t \end{aligned} \tag{A-31}$$

- Capital:

$$\begin{aligned} 1 &= \beta\frac{\lambda_{t+1}^B}{\lambda_t^F}\left[\left(1-1/\theta_f\right) + \frac{\phi_p}{\theta_f}\left(\pi_{t+1}-\bar{\pi}\right)\pi_{t+1} - \frac{\phi_p}{2}\left(\pi_{t+1}-\bar{\pi}\right)^2\right]MPK_{t+1} \\ &\quad -\beta^2\frac{\lambda_{t+2}^B}{\lambda_t^F}\frac{\phi_p}{\theta_f}\left(\pi_{t+2}-\bar{\pi}\right)\pi_{t+2}\frac{Y_{t+2}}{Y_{t+1}}MPK_{t+1} \\ &\quad -\beta\frac{\lambda_{t+1}^B}{\lambda_t^F}\frac{\partial AC_{t+1}^k}{\partial K_t} + \beta\frac{\lambda_{j,t+1}^F}{\lambda_t^F}\left(1-\delta_k\right) \end{aligned} \tag{A-32}$$

- Energy:

$$\begin{aligned} P_t^e &= \left[1-1/\theta_f + \frac{\phi_p}{\theta_f}\left(\pi_t-\bar{\pi}\right)\pi_t - \frac{\phi_p}{2}\left(\pi_t-\bar{\pi}\right)^2\right]MPE_t \\ &\quad -\beta\frac{\lambda_{t+1}^B}{\lambda_t^B}\frac{\phi_p}{\theta_f}\left(\pi_{t+1}-\bar{\pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}MPE_t \end{aligned} \tag{A-33}$$

- Investment:

$$\lambda_t^B\left[1+\frac{\partial AC_t^k}{\partial I_{k,t}}\right] = \lambda_t^F \tag{A-34}$$

- Profit:

$$\Pi_t^f = Y_t - W_tH_tR_t - I_{k,t} - P_t^eE_{f,t} - AC_t^k - AC_t^p \tag{A-35}$$

Equations to be fed into Dynare Since we want all endogenous variables in Dynare to be stationary we define

$$
\begin{aligned}
dp_t &= \frac{DP_t}{P_t} \\
m_t &= \frac{M_t}{P_t} \\
\pi_t &= \frac{P_t}{P_{t-1}}
\end{aligned}
$$

We also plug in for the partial derivatives of the adjustment cost functions. See Appendix D for the derivation.

- Non-Durables:

$$u_1\left(C_t^A, H_t\right)\frac{\partial C_t^A}{\partial N_t} = \lambda_{i,t}^B R_t$$

Thus:

$$\frac{\varphi}{C_t^A}\left(1-\gamma\right)N_t^{-1}C_t^A = \lambda_{i,t}^B R_t$$

or:

$$\lambda_{i,t}^B R_t N_t = \varphi\left(1-\gamma\right) \tag{A-36}$$

- Durables:

$$0 = \beta u_1\left(C_{t+1}^A, H_{t+1}\right)\frac{\partial C_{i,t+1}^A}{\partial D_{i,t}} - \beta\lambda_{t+1}^B\frac{\partial AC_{t+1}^d}{\partial D_t} - \lambda_t^I + \left(1-\delta_d\right)\beta\lambda_{t+1}^I$$

Thus:

$$
\begin{aligned}
\lambda_t^I &= \beta u_1\left(C_{t+1}^A, H_{t+1}\right)\frac{\partial C_{i,t+1}^A}{\partial D_{i,t}} + \left(1-\delta_d\right)\beta\lambda_{t+1}^I - \beta\lambda_{t+1}^B\frac{\partial AC_{t+1}^d}{\partial D_t} \\
&= \beta\varphi\eta_h\gamma\left(\eta_h D_t^{\nu_h} + \left(1-\eta_h\right)E_{h,t+1}^{\nu_h}\right)^{-1}D_t^{\nu_h-1} \\
&\quad + \left(1-\delta_d\right)\beta\lambda_{t+1}^I + \beta\lambda_{t+1}^B\phi_d\left(\frac{I_{d,t+1}}{D_t} - \delta_d\right)\left(\frac{I_{d,t+1}}{D_t}\right)^2
\end{aligned} \tag{A-37}
$$

- Energy:

$$u_1\left(C_t^A, H_t\right)\frac{\partial C_t^A}{\partial E_{h,t}} = P_t^e R_t \lambda_{i,t}^B$$

Thus:

$$P_t^e R_t \lambda_{i,t}^B = \varphi\gamma\left(1-\eta_h\right)\left(\eta_h D_{t-1}^{\nu_h} + \left(1-\eta_h\right)E_{h,t}^{\nu_h}\right)^{-1}E_{h,t}^{\nu_h-1} \tag{A-38}$$

- Durables investment:

$$\lambda_t^B\left[\phi_d\left(\frac{I_{d,t}}{D_{t-1}} - \delta_d\right)\left(\frac{3}{2}\frac{I_{d,t}}{D_{t-1}} - \frac{1}{2}\delta_d\right) + R_t\right] = \lambda_t^I \tag{A-39}$$

- Money holdings and deposits

$$1 = \beta\frac{\lambda_{t+1}^B}{\lambda_{i,t}^B}\frac{R_{t+1}}{\pi_{t+1}} \tag{A-40}$$

- Hours:

$$\begin{aligned}-\beta^t u_2\left(C_t^A, H_t\right) &= \beta^t \lambda_t^B \left(1-\frac{1}{\theta_w}\right) W_t \\ &\quad +\frac{1}{\theta_w}\frac{W_t}{H_t}\left[\beta^t \lambda_{i,t}^B \frac{\partial AC_{i,t}^w}{\partial W_{i,t}} + \beta^{t+1}\lambda_{i,t+1}^B \frac{\partial AC_{i,t+1}^w}{\partial W_{i,t}}\right]\end{aligned}$$

Thus:

$$\begin{aligned}\frac{1-\varphi}{1-H_t} &= \lambda_t^B \left(1-\frac{1}{\theta_w}\right) W_t \\ &\quad +\frac{\phi_w}{\theta_w}\frac{W_t}{H_t}\lambda_{i,t}^B\left[\frac{3}{2}\left(\pi_t \frac{W_t}{W_{t-1}}\right)^2 - 2\pi_t \frac{W_t}{W_{t-1}}\bar{\pi} + \frac{1}{2}\bar{\pi}^2\right] \\ &\quad -\frac{\phi_w}{\theta_w}\beta\frac{W_t}{H_t}\lambda_{i,t+1}^B\left[\left(\pi_{t+1}\frac{W_{t+1}}{W_t} - \bar{\pi}\right)\pi_{t+1}\frac{W_{t+1}^2}{W_t^2}\right] \qquad \text{(A-41)}\end{aligned}$$

- Budget Constraint: Write firm profits as:

$$\begin{aligned}\Pi_t^f &= Y_t - W_t H_t R_t - I_{k,t} - P_t^e E_{f,t} - AC_t^k\left(I_{k,t}, K_{t-1}\right) - AC_t^p\left(P_t, P_{t-1}, Y_t\right) \\ &= N_t + I_{d,t} + P_t^e E_{h,t} + AC_t^w + AC_t^d - W_t H_t R_t\end{aligned}$$

The intermediary's profit is equal to the money injection plus interest. The money injection has to be difference between the amount loaned out to the firm and the household deposits. Thus, we can write the profit as:

$$\Pi_t^b = R_t\left(P_t W_t H_t - DP_t\right)$$

Plug all of this and the cash in advance constraint into the budget constraint:

$$\begin{aligned}m_t &= m_{t-1}\pi_t^{-1} - \left(N_t + I_{d,t} + P_t^e E_{h,t}\right) - dp_t - \left(AC_t^w + AC_t^d\right) + \left(1+R_t\right) W_t H_t \\ &\quad + N_t + I_{d,t} + P_t^e E_{h,t} + AC_t^w + AC_t^d - W_t H_t R_t\end{aligned}$$

Thus

$$m_t = m_{t-1}\pi_t^{-1} - dp_t + W_t H_t \qquad \text{(A-42)}$$

- Cash in advance:

$$N_t + I_{d,t} + P_t^e E_{h,t} = m_{t-1}\pi_t^{-1} - dp_t \qquad \text{(A-43)}$$

- Definition of durables investment:

$$I_{d,t} + \left(1-\delta_d\right) D_{t-1} = D_t \qquad \text{(A-44)}$$

- Consumption aggregator:

$$C_t^A = N_t^{1-\gamma}\left(\eta_h D_{t-1}^{\nu_h} + \left(1-\eta_h\right) E_{h,t}^{\nu_h}\right)^{\frac{\gamma}{\nu_h}} \qquad \text{(A-45)}$$

- Output:

$$Y_t = Z_t H_t^{1-\alpha}\left(\eta_f K_{t-1}^{\nu_f} + \left(1-\eta_f\right) E_{f,t}^{\nu_f}\right)^{\alpha/\nu_f} \qquad \text{(A-46)}$$

- Marginal Product of Labor:

$$MPL_t = (1-\alpha)\frac{Y_t}{H_t} \tag{A-47}$$

- Marginal Product of Capital:

$$MPK_t = Z_t H_t^{1-\alpha}\left(\eta_f K_{t-1}^{\nu_f} + (1-\eta_f)E_{f,t}^{\nu_f}\right)^{\alpha/\nu_f - 1}\alpha\eta_f K_{t-1}^{\nu_f-1} \tag{A-48}$$

- Marginal Product of Energy:

$$MPE_t = Z_t H_t^{1-\alpha}\left(\eta_f K_{t-1}^{\nu_f} + (1-\eta_f)E_{f,t}^{\nu_f}\right)^{\alpha/\nu_f - 1}\alpha(1-\eta_f)E_{f,t}^{\nu_f-1} \tag{A-49}$$

- Hours:

$$\begin{aligned} W_t R_t &= \left[1 - 1/\theta_f + \frac{\phi_p}{\theta_f}(\pi_t - \bar{\pi})\pi_t - \frac{\phi_p}{2}(\pi_t - \bar{\pi})^2\right]MPL_t \\ &\quad -\beta\frac{\lambda_{t+1}^B}{\lambda_t^B}\frac{\phi_p}{\theta_f}(\pi_{t+1} - \bar{\pi})\pi_{t+1}\frac{Y_{t+1}}{Y_t}MPL_t \end{aligned} \tag{A-50}$$

- Capital:

$$\begin{aligned} \lambda_t^F &= \beta\lambda_{t+1}^B\left[(1 - 1/\theta_f) + \frac{\phi_p}{\theta_f}(\pi_{t+1} - \bar{\pi})\pi_{t+1} - \frac{\phi_p}{2}(\pi_{t+1} - \bar{\pi})^2\right]MPK_{t+1} \\ &\quad -\beta^2\frac{\phi_p}{\theta_f}\lambda_{t+2}^B(\pi_{t+2} - \bar{\pi})\pi_{t+2}\frac{Y_{t+2}}{Y_{t+1}}MPK_{t+1} \\ &\quad +\beta\phi_k\lambda_{t+1}^B\left(\frac{I_{k,t+1}}{K_t} - \delta_k\right)\left(\frac{I_{k,t+1}}{K_t}\right)^2 + \beta\lambda_{i,t+1}^F(1-\delta_k) \end{aligned} \tag{A-51}$$

- Capital Law of Motion:

$$K_t = (1-\delta_k)K_{t-1} + I_{k,t} \tag{A-52}$$

- Energy:

$$\begin{aligned} P_t^e &= \left[1 - 1/\theta_f + \frac{\phi_p}{\theta_f}(\pi_t - \bar{\pi})\pi_t - \frac{\phi_p}{2}(\pi_t - \bar{\pi})^2\right]MPE_t \\ &\quad -\beta\frac{\lambda_{t+1}^B}{\lambda_t^B}\frac{\phi_p}{\theta_f}(\pi_{t+1} - \bar{\pi})\pi_{t+1}\frac{Y_{t+1}}{Y_t}MPE_t \end{aligned} \tag{A-53}$$

- Investment:

$$\lambda_t^B\left[1 + \phi_k\left(\frac{I_{k,t}}{K_{t-1}} - \delta_k\right)\left(\frac{3}{2}\frac{I_{k,t}}{K_{t-1}} - \frac{1}{2}\delta_k\right)\right] = \lambda_t^F \tag{A-54}$$

- Aggregate resource constraint:

$$Y_t = N_t + I_{d,t} + I_{k,t} + P_t^e(E_{h,t} + E_{f,t}) + AC_t^w + AC_t^d + AC_t^p + AC_t^k$$

Thus:

$$
\begin{aligned}
Y_t &= N_t + I_{d,t} + I_{k,t} + P_t^e \left(E_{h,t} + E_{f,t}\right) \\
&+ \frac{\phi_w}{2}\left(\pi_t \frac{W_t}{W_{t-1}} - \bar{\pi}\right)^2 W_t \\
&+ \frac{\phi_d}{2}\left(\frac{I_{d,t}}{D_{t-1}} - \delta_d\right)^2 I_{d,t} \\
&+ \frac{\phi_p}{2}\left(\pi_t - \bar{\pi}\right)^2 Y_t \\
&+ \frac{\phi_k}{2}\left(\frac{I_{k,t}}{K_{t-1}} - \delta_k\right)^2 I_{k,t} \qquad \text{(A-55)}
\end{aligned}
$$

- Productivity:

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_{z,t} \qquad \text{(A-56)}$$

- Energy price:

$$\log P_t^e = \rho_e^0 \log P_{t-1}^e + \varepsilon_{p,t} + \rho_e^1 \varepsilon_{p,t-1} \qquad \text{(A-57)}$$

- Money rule:

$$R_t - R = \rho_r \left(R_{t-1} - R\right) + \left(1 - \rho_r\right) \tau_\pi \left(\pi_t - \bar{\pi}\right) + \left(1 - \rho_r\right) \tau_y \left(Y_t - \bar{Y}\right) \qquad \text{(A-58)}$$

We have the following variables:

- 10 consumer variables:

$$C^A, H, N, I_d, E_h, D, m, dp, \lambda^B, \lambda^I$$

- 4 Prices:

$$\pi, R, W, P^e$$

- 9 Production variables:

$$Z, Y, MPL, MPK, MPE, I_k, K, E_f, \lambda^F$$

23 variables and 23 equations

B Construct steady state

Note that in steady state all adjustment costs are zero. Then the first order conditions in steady state are:

- Non-Durables:

$$u_1\left(C^A, H\right) \frac{\partial C^A}{\partial N} = \lambda^B R \qquad \text{(B-1)}$$

- Durables:

$$0 = \beta u_1\left(C^A, H\right) \frac{\partial C^A}{\partial D} - \lambda^I + \left(1 - \delta_d\right) \beta \lambda^I \qquad \text{(B-2)}$$

- Energy:

$$u_1\left(C^A, H\right) \frac{\partial C^A}{\partial E_h} = P^e R \lambda^B \qquad \text{(B-3)}$$

- Investment:

$$\lambda^B R = \lambda^I \tag{B-4}$$

- Money holdings and deposits

$$1 = \beta \frac{R}{\pi} \tag{B-5}$$

- Hours:

$$-u_2\left(C^A, H\right) = \lambda^B \left(1 - \frac{1}{\theta_w}\right) W \tag{B-6}$$

- Budget Constraint

$$m = \frac{m}{\pi} - dp + WH \tag{B-7}$$

- Cash in advance:

$$N + I_d + P^e E_h = \frac{m}{\pi} - dp \tag{B-8}$$

- Durables law of motion:

$$I_d = \delta_d D \tag{B-9}$$

- Consumption aggregator:

$$C^A = N^{1-\gamma} \left(\eta_h D^{\nu_h} + (1-\eta_h) E_h^{\nu_h}\right)^{\frac{\gamma}{\nu_h}} \tag{B-10}$$

- Output:

$$Y = ZH^{1-\alpha} \left(\eta_f K^{\nu_f} + (1-\eta_f) E_f^{\nu_f}\right)^{\alpha/\nu_f} \tag{B-11}$$

- Marginal Product of Labor:

$$MPL = (1-\alpha) \frac{Y}{H} \tag{B-12}$$

- Marginal Product of Capital:

$$MPK = ZH^{1-\alpha} \left(\eta_f K^{\nu_f} + (1-\eta_f) E_f^{\nu_f}\right)^{\alpha/\nu_f - 1} \alpha \eta_f K^{\nu_f - 1} \tag{B-13}$$

- Marginal Product of Energy:

$$MPE = ZH^{1-\alpha} \left(\eta_f K^{\nu_f} + (1-\eta_f) E_f^{\nu_f}\right)^{\alpha/\nu_f - 1} \alpha (1-\eta_f) E_f^{\nu_f - 1} \tag{B-14}$$

- Hours:

$$(1 - 1/\theta_f) MPL = WR \tag{B-15}$$

- Capital:

$$1 = \beta \left[\left(1 - \frac{1}{\theta_f}\right) MPK + 1 - \delta\right] \tag{B-16}$$

- Capital law of motion:

$$I_k = \delta_k K$$

- Energy:

$$(1 - 1/\theta_f) MPE = P^e \tag{B-17}$$

- Investment:

$$\lambda_t^B = \lambda_t^F \tag{B-18}$$

- Aggregate resource constraint:

$$Y = N + I_d + I_k + P^e (E_h + E_f) \tag{B-19}$$

Start cranking From the Capital Euler equation (B-16) and Firm Energy equation (B-17):

$$MPK = \frac{\frac{1}{\beta} - 1 + \delta_k}{1 - \frac{1}{\theta_f}} \tag{B-20}$$

$$MPE = \frac{P^e}{1 - \frac{1}{\theta_f}} \tag{B-21}$$

Also:

$$\frac{MPK}{MPE} = \frac{\eta_f}{(1-\eta_f)} \left(\frac{K}{E_f}\right)^{\nu_f - 1} \tag{B-22}$$

Thus the capital energy ratio is

$$\kappa_{kef} \equiv \frac{K}{E_f} = \left(\frac{MPK}{MPE}\frac{1-\eta_f}{\eta_f}\right)^{\frac{1}{\nu_f - 1}} \tag{B-23}$$

Also from the definition of MPE, equation (B-14)

$$\begin{aligned} MPE &= ZH^{1-\alpha}\left(\eta_f K^{\nu_f} + (1-\eta_f) E_f^{\nu_f}\right)^{\alpha/\nu_f - 1} \alpha (1-\eta_f) E_f^{\nu_f - 1} \\ &= Y\left(\eta_f K^{\nu_f} + (1-\eta_f) E_f^{\nu_f}\right)^{-1} \alpha (1-\eta_f) E_f^{\nu_f - 1} \\ &= \left(\eta_f \frac{(E_f \kappa_{kef})^{\nu_f} E_f^{1-\nu_f}}{Y} + (1-\eta_f)\frac{E_f}{Y}\right)^{-1} \alpha (1-\eta_f) \\ &= \left(\eta_f \frac{(E_f \kappa_{kef})^{\nu_f} E_f^{1-\nu_f}}{Y} + (1-\eta_f)\frac{E_f}{Y}\right)^{-1} \alpha (1-\eta_f) \end{aligned} \tag{B-24}$$

Call $\kappa_{Ef} \equiv \frac{E_f}{Y}$ the firm energy use to output ratio. Then

$$\begin{aligned} MPE &= \left(\eta_f \kappa_{kef}^{\nu_f} \kappa_{Ef} + (1-\eta_f)\kappa_{Ef}\right)^{-1} \alpha (1-\eta_f) \\ &= \frac{\left(\eta_f \kappa_{kef}^{\nu_f} + (1-\eta_f)\right)^{-1} \alpha (1-\eta_f)}{\kappa_{Ef}} \end{aligned} \tag{B-25}$$

or

$$\kappa_{Ef} = \frac{\alpha (1-\eta_f)}{MPE\left(\eta_f \kappa_{kef}^{\nu_f} + 1 - \eta_f\right)} \tag{B-26}$$

Also notice that

$$\kappa_K \equiv \frac{K}{Y} = \frac{K}{E_f}\frac{E_f}{Y} = \kappa_{kef}\kappa_{Ef} \tag{B-27}$$

Next write the output equation (B-11) as:

$$1 = Z\kappa_H^{1-\alpha}\left(\eta_f^{\nu_f}\kappa_K + (1-\eta_f)\kappa_{Ef}^{\nu_f}\right)^{\alpha/\nu_f} \tag{B-28}$$

Thus:

$$\kappa_H = Z^{-\frac{1}{1-\alpha}} \left(\eta_f \kappa_K^{\nu_f} + (1-\eta_f)\,\kappa_{Ef}^{\nu_f}\right)^{-\frac{\alpha}{\nu_f(1-\alpha)}} \tag{B-29}$$

Then through equations (B-12) and (B-15) we get the steady state wage

$$W = \frac{(1-1/\theta_f)(1-\alpha)}{R}\kappa_H^{-1} \tag{B-30}$$

From equations (B-1) and (B-3)

$$\begin{aligned}
P^e &= \frac{\frac{\partial C^A}{\partial E_h}}{\frac{\partial C^A}{\partial N}} \\
&= \frac{\frac{\gamma}{\nu_h} N^{1-\gamma} \left(\eta_h D^{\nu_h} + (1-\eta_h)\,E_h^{\nu_h}\right)^{\frac{\gamma}{\nu_h}-1} (1-\eta_h)\,\nu_h E_h^{\nu_h-1}}{(1-\gamma)\,N^{-\gamma}\left(\eta_h D^{\nu_h} + (1-\eta_h)\,E_h^{\nu_h}\right)^{\frac{\gamma}{\nu_h}}} \\
&= N\frac{\gamma(1-\eta_h)}{1-\gamma} E_h^{\nu_h-1}\gamma\left(\eta_h D^{\nu_h} + (1-\eta_h)\,E_h^{\nu_h}\right)^{-1} \qquad \text{(B-31)}
\end{aligned}$$

Thus:

$$\left(\eta_h D^{\nu_h} + (1-\eta_h)\,E_h^{\nu_h}\right)^{-1} = P^e \frac{1-\gamma}{\gamma(1-\eta_h)} E_h^{1-\nu_h} N^{-1} \tag{B-32}$$

From the durables vs. non-durables equation (equations (B-2) and (B-1)):

$$\begin{aligned}
1 &= \beta\frac{\frac{\partial C^A}{\partial D}}{\frac{\partial C^A}{\partial N}} + (1-\delta_d)\,\beta \\
&= \beta\frac{\gamma N^{1-\gamma}\left(\eta_h D^{\nu_h} + (1-\eta_h)\,E_h^{\nu_h}\right)^{\frac{\gamma}{\nu_h}-1}\eta_h D^{\nu_h-1}}{(1-\gamma)\,N^{-\gamma}\left(\eta_h D^{\nu_h} + (1-\eta_h)\,E_h^{\nu_h}\right)^{\frac{\gamma}{\nu_h}}} + (1-\delta_d)\,\beta \\
&= \beta N\frac{\gamma\eta_h}{1-\gamma}\left(\eta_h D^{\nu_h} + (1-\eta_h)\,E_h^{\nu_h}\right)^{-1} D^{\nu_h-1} + (1-\delta_d)\,\beta \\
&= \beta N\frac{\gamma\eta_h}{1-\gamma} P^e \frac{1-\gamma}{\gamma(1-\eta_h)} E_h^{1-\nu_h} N^{-1} D^{\nu_h-1} + (1-\delta_d)\,\beta \\
&= \beta N\frac{\gamma\eta_h}{1-\gamma} P^e \frac{1-\gamma}{\gamma(1-\eta_h)} E_h^{1-\nu_h} N^{-1} D^{\nu_h-1} + (1-\delta_d)\,\beta \qquad \text{(B-33)}
\end{aligned}$$

by plugging in for $\left(\eta_h D^{\nu_h} + (1-\eta_h)\,E_h^{\nu_h}\right)^{-1}$. Then

$$1 = \beta\frac{\eta_h}{1-\eta_h} P^e \left(\frac{E_h}{D}\right)^{1-\nu_h} + (1-\delta_d)\,\beta \tag{B-34}$$

Thus:

$$\frac{E_h}{D} = \left[\frac{1-\beta+\beta\delta_d}{\beta\eta_h P_e}(1-\eta_h)\right]^{1/(1-\nu_h)} \tag{B-35}$$

Next, rewrite the nondurables and energy equation as:

$$\begin{aligned} P^e &= N\frac{\gamma(1-\eta_h)}{1-\gamma}E_h^{\nu_h-1}\gamma\left(\eta_h D^{\nu_h}+(1-\eta_h)E_h^{\nu_h}\right)^{-1} \\ &= \frac{N}{D}\frac{\gamma(1-\eta_h)}{1-\gamma}\left(\frac{E_h}{D}\right)^{\nu_h-1}\gamma\left(\eta_h+(1-\eta_h)\left(\frac{E_h}{D}\right)^{\nu_h}\right)^{-1} \\ &= \frac{N}{D}\frac{\gamma(1-\eta_h)}{1-\gamma}\left(\eta_h\left(\frac{E_h}{D}\right)^{1-\nu_h}+(1-\eta_h)\frac{E_h}{D}\right)^{-1} \end{aligned} \tag{B-36}$$

Thus:

$$\frac{N}{D}=\frac{(1-\gamma)P^e}{\gamma(1-\eta_h)}\left(\eta_h\left(\frac{E_h}{D}\right)^{1-\nu_h}+(1-\eta_h)\frac{E_h}{D}\right) \tag{B-37}$$

Then rewrite the resource constraint (B-19) as:

$$\begin{aligned} 1 &= \frac{N}{Y}+\frac{I_d}{Y}+\frac{I_k}{Y}+P^e\left(\frac{E_h}{Y}+\frac{E_f}{Y}\right) \\ &= \delta\kappa_k+P^e\kappa_{Ef}+\kappa_D\left[\frac{N}{D}+\delta_d+P^e\frac{E_h}{D}\right] \end{aligned} \tag{B-38}$$

Then:

$$\kappa_D=\frac{1-\delta_k\kappa_k-P^e\kappa_{Ef}}{\frac{N}{D}+\delta_d+P^e\frac{E_h}{D}} \tag{B-39}$$

Notice that so far we have not used a particular functional form for the utility function. Now we can calculate $\kappa_N=\frac{N}{Y}$ as

$$\kappa_N=\frac{N}{D}\kappa_D \tag{B-40}$$

On the consumer side there is one more equation left over:

$$-u_2\left(C^A,H\right)=\frac{u_1\left(C^A,H\right)}{R}\frac{\partial C^A}{\partial N}\left(1-\frac{1}{\theta_w}\right)W \tag{B-41}$$

Assume that u takes the form:

$$u\left(C^A,H\right)=\varphi\log C^A+(1-\varphi)\log(1-H) \tag{B-42}$$

Then:

$$u_1\left(C^A,H\right)\frac{\partial C^A}{\partial N}=\varphi(1-\gamma)N^{-1} \tag{B-43}$$

Thus:

$$\frac{1-\varphi}{1-H}=\frac{\varphi(1-\gamma)}{R}N^{-1}\left(1-\frac{1}{\theta_w}\right)W \tag{B-44}$$

Also:

$$WR=\left(1-\frac{1}{\theta_f}\right)(1-\alpha)\frac{Y}{H} \tag{B-45}$$

Then:

$$\frac{1-\varphi}{1-H}=\frac{\varphi(1-\gamma)}{R^2}N^{-1}\left(1-\frac{1}{\theta_w}\right)\left(1-\frac{1}{\theta_f}\right)(1-\alpha)\frac{Y}{H} \tag{B-46}$$

Then

$$\frac{1-H}{H} = \frac{1-\varphi}{\varphi} R^2 \kappa_N \left[(1-\gamma)(1-\alpha)\left(1-\frac{1}{\theta_w}\right)\left(1-\frac{1}{\theta_f}\right)\right]^{-1} \tag{B-47}$$

Solve for H :

$$H = \frac{1}{1+\frac{1-\varphi}{\varphi} R^2 \kappa_N \left[(1-\gamma)(1-\alpha)\left(1-\frac{1}{\theta_w}\right)\left(1-\frac{1}{\theta_f}\right)\right]^{-1}} \tag{B-48}$$

Next compute

$$Y = \frac{H}{\kappa_H} \tag{B-49}$$

Also, from the money/deposits Euler equation

$$\pi = \beta R \tag{B-50}$$

and from there all the other variables via their output ratios. We compute real deposits and real money holdings via:

$$m = N + I_d + P^e E_h + WH \tag{B-51}$$

$$dp = \frac{m}{\pi} - m + WH \tag{B-52}$$

Finally, the Lagrange Multipliers in steady state:

$$\lambda^B = \frac{\varphi(1-\gamma)}{RN} \tag{B-53}$$

$$\lambda^I = R\lambda^B \tag{B-54}$$

C Calibration

We set targets for steady state values of ratios E_h/Y, I_d/Y, D/Y, E_f/Y, K/Y and hours worked H. We use these targets to pin down six parameters $\eta_h, \gamma, \eta_f, \delta_d, \delta_k, \varphi$.

From the firm energy use equation (B-17):

$$\begin{aligned} P^e &= (1-1/\theta_f)\, Y \left(\eta_f K^{\nu_f} + (1-\eta_f) E_f^{\nu_f}\right)^{-1} \alpha (1-\eta_f) E_f^{\nu_f - 1} \\ &= (1-1/\theta_f)\frac{Y}{E_f}\left(\eta_f\left(\frac{K}{E_f}\right)^{\nu_f} + (1-\eta_f)\right)^{-1}\alpha(1-\eta_f) \\ &= (1-1/\theta_f)\left(\frac{K}{Y}\right)^{-1}\frac{K}{E_f}\left(\eta_f \kappa_{Ef}^{\nu_f} + (1-\eta_f)\right)^{-1}\alpha(1-\eta_f) \end{aligned} \tag{C-1}$$

This equation pins down η_f. This is the same root finding problem as in Dhawan and Jeske (2006) but with the additional factor $(1-1/\theta_f)$.

Next, we find the steady state MPK:

$$\begin{aligned} MPK &= Y\left(\eta_f K^{\nu_f} + (1-\eta_f) E_f^{\nu_f}\right)^{-1}\alpha\eta_f K^{\nu_f-1} \\ &= \left(\frac{K}{Y}\right)^{-1}\left(\eta_f + (1-\eta_f)\left(\frac{K}{E_f}\right)^{-\nu_f}\right)^{-1}\alpha\eta_f \end{aligned} \tag{C-2}$$

and from capital Euler equation (B-16) we solve for δ_k :

$$\delta_k = \left(1 - \frac{1}{\theta_f}\right) MPK - \frac{1}{\beta} + 1 \tag{C-3}$$

On the household side we can easily determine δ_d from the calibration targets

$$\delta_d = \frac{I_d/Y}{D/Y} \tag{C-4}$$

The durables Euler equation:

$$\begin{aligned} 1 &= \beta \frac{u_1\left(C^A, H\right) \frac{\partial C^A}{\partial D}}{u_1\left(C^A, H\right) \frac{\partial C^A}{\partial N}} + (1 - \delta_d)\beta \\ &= \beta\eta_h \frac{\gamma}{1-\gamma} N \left(\eta_h D^{\nu_h} + (1-\eta_h) E_h^{\nu_h}\right)^{-1} D^{\nu_h - 1} + (1-\delta_d)\beta \end{aligned} \tag{C-5}$$

From energy vs. nondurables:

$$P^e = N \frac{\gamma}{1-\gamma} (1-\eta_h) E_h^{\nu_h - 1} \left(\eta_h D^{\nu_h} + (1-\eta_h) E_h^{\nu_h}\right)^{-1} \tag{C-6}$$

Solve for $\frac{\gamma}{1-\gamma}$:

$$\frac{\gamma}{1-\gamma} = P^e N^{-1} (1-\eta_h)^{-1} E_h^{1-\nu_h} \left(\eta_h D^{\nu_h} + (1-\eta_h) E_h^{\nu_h}\right) \tag{C-7}$$

and plug into equation (C-5):

$$1 = \beta \frac{\eta_h}{1-\eta_h} P^e \left(\frac{D}{E_h}\right)^{\nu_h - 1} + (1-\delta_d)\beta \tag{C-8}$$

Solve for η_h :

$$\frac{1-\eta_h}{\eta_h} = \frac{\beta P^e \left(\frac{D}{E_h}\right)^{\nu_h - 1}}{1 - (1-\delta_d)\beta} \tag{C-9}$$

Thus:

$$\eta_h = \frac{1 - \beta(1-\delta_d)}{1 - (1-\delta_d)\beta + \beta P^e \left(\frac{D}{E_h}\right)^{\nu_h - 1}} \tag{C-10}$$

In equation (C-7) we solve for γ :

$$\frac{\gamma}{1-\gamma} = P^e \left(\frac{E_h}{N}\right) (1-\eta_h)^{-1} \left(\eta_h \left(\frac{D}{E_h}\right)^{\nu_h} + 1 - \eta_h\right) \tag{C-11}$$

Thus:

$$\begin{aligned} \gamma &= 1 - \frac{1}{1 + P^e \left(\frac{E_h}{N}\right) (1-\eta_h)^{-1} \left(\eta_h \left(\frac{D}{E_h}\right)^{\nu_h} + 1 - \eta_h\right)} \\ &= 1 - \frac{1-\eta_h}{1 - \eta_h + P^e \left(\frac{E_h}{N}\right) \left(\eta_h \left(\frac{D}{E_h}\right)^{\nu_h} + 1 - \eta_h\right)} \end{aligned} \tag{C-12}$$

Finally, combine the labor supply equation:

$$\frac{1-\varphi}{1-H} = \frac{\varphi\left(1-\gamma\right)}{R} N^{-1}\left(1-\frac{1}{\theta_w}\right) W \tag{C-13}$$

and

$$WR = \left(1-\frac{1}{\theta_f}\right)\left(1-\alpha\right)\frac{Y}{H} \tag{C-14}$$

to get:

$$\frac{1-\varphi}{\varphi} = \frac{1-H}{H}\left(1-\gamma\right)\frac{\left(1-\frac{1}{\theta_w}\right)\left(1-\frac{1}{\theta_f}\right)}{R^2}\left(1-\alpha\right)\frac{Y}{N} \tag{C-15}$$

Thus:

$$\varphi = \left[1+\frac{1-H}{H}\left(1-\gamma\right)\frac{\left(1-\frac{1}{\theta_w}\right)\left(1-\frac{1}{\theta_f}\right)}{R^2}\left(1-\alpha\right)\frac{Y}{N}\right]^{-1} \tag{C-16}$$

where

$$\frac{Y}{N} = \left(\frac{D}{Y}\right)^{-1}\frac{I_d}{N}\delta_d^{-1} \tag{C-17}$$

D Partial Derivatives of adjustment cost functions

Drop the i and j subscripts to simplify notation

$$AC_t^w\left(W_t, W_{t-1}\right) = \frac{\phi_w}{2}\left(\frac{P_t W_t}{P_{t-1}W_{t-1}}-\bar{\pi}\right)^2 W_t \tag{D-1}$$

and the partial derivatives are:

$$\begin{aligned}\frac{\partial AC_t^w\left(W_t, W_{t-1}\right)}{\partial W_t} &= \phi_w\left(\frac{P_t W_t}{P_{t-1}W_{t-1}}-\bar{\pi}\right)\frac{P_t W_t}{P_{t-1}W_{t-1}}+\frac{\phi_w}{2}\left(\frac{P_t W_t}{P_{t-1}W_{t-1}}-\bar{\pi}\right)^2 \\ &= \left(\frac{P_t W_t}{P_{t-1}W_{t-1}}\right)^2\left[\phi_w+\frac{\phi_w}{2}\right]+\frac{P_t W_t}{P_{t-1}W_{t-1}}\bar{\pi}\left[-\phi_w-\phi_w\right] \\ &\quad +\frac{\phi_w}{2}\bar{\pi}^2 \\ &= \frac{3\phi_w}{2}\left(\pi_t\frac{W_t}{W_{t-1}}\right)^2-2\phi_w\pi_t\frac{W_t}{W_{t-1}}\bar{\pi}+\frac{\phi_w}{2}\bar{\pi}^2\end{aligned} \tag{D-2}$$

and

$$\frac{\partial AC_t^w\left(W_t, W_{t-1}\right)}{\partial W_{t-1}} = -\phi_w\left(\pi_t\frac{W_t}{W_{t-1}}-\bar{\pi}\right)\pi_t\frac{W_t^2}{W_{t-1}^2} \tag{D-3}$$

Adjustment cost for durables investment:

$$AC_t^d\left(I_{d,t}, D_{t-1}\right) = \frac{\phi_d}{2}\left(\frac{I_{d,t}}{D_{t-1}}-\delta_d\right)^2 I_{d,t} \tag{D-4}$$

Thus

$$\begin{aligned}\frac{\partial AC_t^d\left(I_{d,t},D_{t-1}\right)}{\partial I_{d,t}} &= \phi_d\left(\frac{I_{d,t}}{D_{t-1}}-\delta_d\right)\frac{I_{d,t}}{D_{t-1}}+\frac{\phi_d}{2}\left(\frac{I_{d,t}}{D_{t-1}}-\delta_d\right)^2\\ &= \phi_d\left(\frac{I_{d,t}}{D_{t-1}}-\delta_d\right)\left[\frac{I_{d,t}}{D_{t-1}}+\frac{1}{2}\left(\frac{I_{d,t}}{D_{t-1}}-\delta_d\right)\right]\\ &= \phi_d\left(\frac{I_{d,t}}{D_{t-1}}-\delta_d\right)\left[\frac{3}{2}\frac{I_{d,t}}{D_{t-1}}-\frac{1}{2}\delta_d\right] \end{aligned}\tag{D-5}$$

and

$$\frac{\partial AC_t^d\left(I_{d,t},D_{t-1}\right)}{\partial D_{t-1}} = -\phi_d\left(\frac{I_{d,t}}{D_{t-1}}-\delta_d\right)\left(\frac{I_{d,t}}{D_{t-1}}\right)^2\tag{D-6}$$

On the firm side, again drop the j subscripts to simplify notation

$$AC_t^k\left(I_{k,t},K_{t-1}\right)=\frac{\phi_k}{2}\left(\frac{I_{k,t}}{K_{t-1}}-\delta_k\right)^2 I_{k,t}\tag{D-7}$$

Then:

$$\begin{aligned}\frac{\partial AC_t^k\left(I_{k,t},K_{t-1}\right)}{\partial I_{k,t}} &= \phi_k\left(\frac{I_{k,t}}{K_{t-1}}-\delta_k\right)\frac{I_{k,t}}{K_{t-1}}+\frac{\phi_k}{2}\left(\frac{I_{k,t}}{K_{t-1}}-\delta_k\right)^2\\ &= \phi_k\left(\frac{I_{k,t}}{K_{t-1}}-\delta_k\right)\left[\frac{I_{k,t}}{K_{t-1}}+\frac{1}{2}\left(\frac{I_{k,t}}{K_{t-1}}-\delta_k\right)\right]\\ &= \phi_k\left(\frac{I_{k,t}}{K_{t-1}}-\delta_k\right)\left[\frac{3}{2}\frac{I_{k,t}}{K_{t-1}}-\frac{1}{2}\delta_k\right] \end{aligned}\tag{D-8}$$

and

$$\frac{\partial AC_t^k\left(I_{k,t},K_{t-1}\right)}{\partial K_{t-1}} = -\phi_k\left(\frac{I_{k,t}}{K_{t-1}}-\delta_k\right)\left(\frac{I_{k,t}}{K_{t-1}}\right)^2\tag{D-9}$$

Finally, adjustment costs for the nominal price of the intermediary are:

$$AC_t^p\left(P_t,P_{t-1},Y_t\right)=\frac{\phi_p}{2}\left(\frac{P_t}{P_{t-1}}-\bar{\pi}\right)^2 Y_t\tag{D-10}$$

Then:

$$\begin{aligned}\frac{\partial AC_t^p\left(P_t,P_{t-1},Y_t\right)}{\partial P_t} &= \phi_p\left(\frac{P_t}{P_{t-1}}-\bar{\pi}\right)\frac{Y_t}{P_{t-1}}\\ \frac{\partial AC_t^p\left(P_t,P_{t-1},Y_t\right)}{\partial P_{t-1}} &= -\phi_p\left(\frac{P_t}{P_{t-1}}-\bar{\pi}\right)\frac{P_t}{P_{t-1}^2}Y_t\\ \frac{\partial AC_t^p\left(P_t,P_{t-1},Y_t\right)}{\partial Y_t} &= \frac{\phi_p}{2}\left(\frac{P_t}{P_{t-1}}-\bar{\pi}\right)^2 \end{aligned}\tag{D-11}$$

The derivative of prices with respect to output:

$$\frac{\partial P_{j,t}}{\partial Y_{j,t}}=-\frac{1}{\theta_f}P_t Y_{j,t}^{-1-1/\theta_f}Y_t^{1/\theta_f}\tag{D-12}$$

In a symmetric equilibrium:

$$\frac{\partial P_{j,t}}{\partial Y_{j,t}} = -\frac{1}{\theta_f}\frac{P_t}{Y_t} \tag{D-13}$$

The partial derivatives of adjustment cost with respect to production factors:

$$\begin{aligned}\frac{\partial AC_t^p}{\partial H_t} &= \left[\frac{\partial AC_t^p}{\partial P_t}\frac{\partial P_t}{\partial Y_t} + \frac{\partial AC_t^p}{\partial Y_t}\right] MPL_t \\ &= \left[-\phi_p\left(\frac{P_t}{P_{t-1}} - \bar{\pi}\right)\frac{Y_t}{P_{t-1}}\frac{1}{\theta_f}\frac{P_t}{Y_t} + \frac{\phi_p}{2}\left(\frac{P_t}{P_{t-1}} - \bar{\pi}\right)^2\right] MPL_t \\ &= \left[-\frac{\phi_p}{\theta_f}\left(\pi_t - \bar{\pi}\right)\pi_t + \frac{\phi_p}{2}\left(\pi_t - \bar{\pi}\right)^2\right] MPL_t \end{aligned} \tag{D-14}$$

Likewise:

$$\frac{\partial AC_t^p}{\partial E_{f,t}} = \left[-\frac{\phi_p}{\theta_f}\left(\pi_t - \bar{\pi}\right)\pi_t + \frac{\phi_p}{2}\left(\pi_t - \bar{\pi}\right)^2\right] MPE_t \tag{D-15}$$

and

$$\frac{\partial AC_{t+1}^p}{\partial K_t} = \left[-\frac{\phi_p}{\theta_f}\left(\pi_{t+1} - \bar{\pi}\right)\pi_{t+1} + \frac{\phi_p}{2}\left(\pi_{t+1} - \bar{\pi}\right)^2\right] MPK_{t+1} \tag{D-16}$$

And now for leading adjustment costs:

$$\begin{aligned}\frac{\partial AC_{t+1}^p}{\partial H_t} &= \frac{\partial AC_{t+1}^p}{\partial P_t}\frac{\partial P_t}{\partial Y_t} MPL_t \\ &= \phi_p\left(\frac{P_{t+1}}{P_t} - \bar{\pi}\right)\frac{P_{t+1}}{P_t^2} Y_{t+1}\frac{1}{\theta_f}\frac{P_t}{Y_t} MPL_t \\ &= \frac{\phi_p}{\theta_f}\left(\pi_{t+1} - \bar{\pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t} MPL_t \end{aligned} \tag{D-17}$$

Likewise:

$$\frac{\partial AC_{t+1}^p}{\partial E_{f,t}} = \frac{\phi_p}{\theta_f}\left(\pi_{t+1} - \bar{\pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t} MPE_t \tag{D-18}$$

And finally for capital:

$$\begin{aligned}\frac{\partial AC_{t+2}^p}{\partial K_t} &= \frac{\partial AC_{t+2}^p}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial Y_{t+1}} MPK_{t+1} \\ &= \phi_p\left(\frac{P_{t+2}}{P_{t+1}} - \bar{\pi}\right)\frac{P_{t+2}}{P_{t+1}^2} Y_{t+2}\frac{1}{\theta_f}\frac{P_{t+1}}{Y_{t+1}} MPK_{t+1} \\ &= \frac{\phi_p}{\theta_f}\left(\pi_{t+2} - \bar{\pi}\right)\pi_{t+2}\frac{Y_{t+2}}{Y_{t+1}} MPK_{t+1} \end{aligned} \tag{D-19}$$